Quantifying Informational Illiquidity in Corporate Bond Markets*

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Abstract

We develop a credit-risk model to quantify the effect of information asymmetry on corporate bond pricing. Bondholders sell their bonds in secondary markets either to meet liquidity needs or to exploit informational advantages. The bond price is determined by the interaction between informed selling and liquidity-driven sellers' efforts to reveal their true selling motives. Consistent with empirical patterns, the model generates a non-monotonic relationship between yield spreads and turnover rates. In the calibrated model, information asymmetry depresses bond prices by 0.51%-1.14% for investment-grade bonds and 2.11%-3.62% for speculative-grade bonds. The model offers several new policy implications regarding bond market liquidity.

Keywords: corporate bonds, liquidity, information asymmetry, turnover rates

JEL Classifications: G33, G14, G12

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1 Introduction

Liquidity plays a crucial role in determining corporate bond prices, as vividly observed during the 2008 financial crisis and the 2020 COVID-19 crisis. For instance, Dick-Nielsen et al. (2012) and Haddad et al. (2021) find a sharp increase in the liquidity component of corporate bond yield spreads at the onsets of both the crisis periods. Friewald et al. (2012), Falato et al. (2021), Kargar et al. (2021), and O'Hara and Zhou (2021) also provide empirical evidence that highlights the vulnerability of corporate bond markets to liquidity, especially during crisis periods.

Given the importance of bond market liquidity, researchers have intensively studied to measure the effect of illiquidity on corporate bond pricing. Specifically, Huang and Huang (2012) first show that a broad class of standard structural credit-risk models, which omit liquidity channels, largely underestimate credit spreads in US corporate bond markets. Expanding this result, Huang et al. (2023b) show that this so-called credit spread puzzle is prevalent across global corporate bond markets. Moreover, He and Xiong (2012), He and Milbradt (2014), Chen et al. (2018), and Huang et al. (2023a) develop structural models to decompose yield spreads into the credit risk and liquidity components systematically, particularly emphasizing the role of debt maturity and rollover risk.

These existing models, however, primarily focus on non-informational frictions, such as search frictions and inventory costs, without explicitly considering illiquidity caused by informational frictions. Thus, our understanding of illiquidity in corporate bond markets remains limited, as informational frictions are commonly recognized as important sources of bond market illiquidity. For instance, Han and Zhou (2014) show that information asymmetry among bond investors significantly influences corporate bond yield spreads, using market microstructure measures. This gap in our understanding of bond market liquidity underscores the need for a structural model that incorporates both informational and non-informational frictions to examine not only their respective roles but also their interactions in shaping overall liquidity in bond markets.

Motivated by this observation, we develop a structural credit-risk model in which the secondary bond market suffers from information asymmetry among bond investors. Specifically, the model assumes that current bondholders have superior information about their firms than potential bond buyers. The informational advantage of current bondholders leads to adverse selection in the secondary bond market, resulting in an informational discount in the equilibrium bond price. The informational discount depresses the price at which bondholders sell their positions for non-informational reasons, thereby causing market illiquidity. We further assume that these bondholders have a costly device to mitigate the adverse selection problem by revealing their trading motives, which will be detailed later.

The main results of our model, which integrates both information asymmetry and the adverse-selection mitigation option, are two folds. First, our model generates three distinct patterns in the relationship between credit risks and the average turnover rates of corporate bonds, namely, an increasing pattern, a hump-shaped pattern, and a hump-shaped pattern followed by a rebound, which we hereafter call the hump-and-rebound pattern. To identify which pattern is most consistent with data, we calibrate the model by matching the cross-sectional relationship between bond yield spreads and turnover rates observed in data. The calibrated result indicates that the hump-and-rebound pattern best aligns with data, as will be illustrated in Figure 4 in more detail later.

Second, our model enables us to disentangle the respective contributions of informational frictions and non-informational frictions to overall illiquidity in bond markets, while existing structural credit-risk models do not allow for this more refined decomposition. Based on the calibrated model parameters, we find that the price discount attributable to informational illiquidity ranges from 0.51% to 1.14% for investment-grade bonds and from 2.11% to 3.62% for high-yield bonds, meaning that informational frictions cause non-negligible effects on investment-grade bonds and significant effects on high-yield bonds. Meanwhile, the price discount caused by non-informational frictions ranges from 4.37% to 4.67% for investment-grade bonds and from 5.42% to 6.85% for high-yield bonds. Qualitatively, these results show that both informational and non-informational liquidity factors cause larger downward price pressures as credit risk rises, which aligns with our intuition, while informational frictions have a qualitatively significant impact even on investment-grade bonds.

We now describe the details of our model. The model considers firms that generate stochastic cash flows over time. Each firm has a fixed amount of bonds diversely held by bond investors. The firm's bonds are traded in a secondary market in which investors face information asymmetry. Specifically, the firm's liquidation value in bankruptcy depends on its intrinsic type, which can be either high or low. We assume that bond sellers are better informed of their firm's future liquidation value than potential bond buyers. As pointed out by Han and Zhou (2014), this assumption is reasonable because of the institutional nature of corporate bond markets: current bondholders are entitled to attend meetings arranged by the firm's management and request inside information about the firm. While bond buyers may also possess some inside information about firms in practice, the assumption that current bondholders have superior information compared to bond buyers is still reasonable because, on average, information asymmetry in corporate bond markets drives down bond prices rather than pushing up the prices, according to Han and Zhou (2014).

In the model, bondholders sell their bonds for two reasons. First, each bondholder is exposed to an idiosyncratic liquidity shock. Once hit by a liquidity shock, the affected bondholder immediately sells her bond holding due to substantially large bond-holding costs. Second, non-liquidity-shocked bondholders may have incentives to sell their bonds to exploit their informational advantage against potential bond buyers. In equilibrium, only non-liquidity-shocked bondholders of low-type firms choose to sell their bond positions for this informational reason. As a result, bond prices are pushed down to some extent, a phenomenon commonly called adverse selection.

Besides adverse selection, we assume that bond sellers face additional trading costs arising from non-informational frictions, which we broadly interpret as search costs, inventory costs, monopoly power of dealers, and so on. This assumption rules out the unrealistic outcome in which all non-liquidity-shocked bondholders of low-type firms sell their bonds at every point in time.

Our model further introduces an adverse-selection mitigation device available to bond sellers, as mentioned above. Specifically, in the model, each bond seller has an option to reveal her liquidity status at costs before selling her bond. This liquidity-status revealing option can be interpreted as a bond seller's costly effort to search for potential bond buyers or dealers who can correctly identify the seller's trading motive based on her balance sheet or trading record. Due to this option, those liquidity-shocked bond sellers who reveal their liquidity status can avoid adverse selection by differentiating themselves against non-liquidity-shocked bond sellers of low-type firms, who sell their bonds for the informational reason.

As anecdotal evidence supporting this assumption, Da et al. (2011) document that many institutional investors seeking to sell securities for non-informational reasons reached out to funds managed by Dimensional Fund Advisors and successfully offloaded their securities without incurring informational liquidity discounts by verifying their true trading motives. Moreover, our assumption is conceptually similar the assumption adopted by Lee and Wang (2024), who consider a model where a buyer can observe whether a seller is informed or not with some noise, because this assumption also allows the buyer to partially identify the seller's true trading motive. For completeness, we also develop an alternative model in which bond sellers have an option to directly disclosure the true type of their firms and show that this model yields qualitatively similar results to those of our main model.

Our model can generate various patterns in the relationship between credit risk and the average turnover rates of bonds. Specifically, in our model, as credit risk rises, the turnover rate may follow an increasing pattern, a hump-shaped pattern, or a hump-and-rebound pattern. More precisely, in all the three cases, the turnover rate does not increase with credit risk when credit risk is low. However, when credit risk is moderate or high, the turnover rate can exhibit one of the aforementioned three patterns as credit risk rises. These diverse patterns emerge in our model because we account for not only information asymmetry but also the adverse-selection mitigation channel.

To understand why distinct patterns may arise in our model, note first that, in our model, the average turnover rate and the size of informed selling exhibit the same pattern, as the number of liquidity-shocked bond sellers is assumed to be constant regardless of the current credit-risk level. Hence, to understand the patterns of turnover rates, we only need to examine the relationship between the firm's credit risk and the equilibrium amount of informed bond selling.

When the firm's credit risk is low, the private information about the firm's future liquidation value has little value. Hence, in this case, non-liquidity-shocked bondholders of low-type firms would have no incentives to sell their bonds for the informational reason, due to the presence of additional trading costs. Thus, the turnover rate does not change in response to an increase in credit risk, in the case of low credit risk.

When the firm's credit risk is moderate, the non-liquidity-shocked bondholders of low-type firms would have some incentives to exploit their private information. Thus, in this case, an increase in credit risk leads to more informed bond selling. However, the size of informed bond selling may not keep increasing in credit risk. Specifically, when credit risk rises further, liquidity-shocked bond sellers will reveal their true trading motive more aggressively. Then, non-liquidity-shocked bond sellers cannot sell their bonds for the informational reason as aggressively as before, because the size of the pool of bond sellers whom they can mimic has declined. We show that this outcome generally arises when adverse-selection mitigation costs are mild. In this case, the relationship between credit risk and turnover rates exhibits a hump-shaped pattern at least when the firm's credit risk is in an intermediate range. In contrast, when adverse-selection mitigation costs are large, the turnover rate keeps increasing in credit risk as a result of increased informed selling, forming the increasing pattern.

When the firm's credit risk is high, liquidity-shocked bondholders have strong incentives to reveal their liquidity status if adverse-selection mitigation costs are mild. However, we assume that the capacity with which liquidity-shocked bond sellers can reveal their liquidity status is limited. This assumption is reasonable because the outcome in which all liquidity-shocked bond sellers reveal their liquidity status at the same time is not realistic. As such, if this maximum capacity is sufficiently large so that the number of liquidity-shocked bond sellers revealing their true trading motive never reaches the maximum possible level, the size of informed bond selling will continue to decrease in credit risk, forming the hump-shaped pattern. However, when this maximum capacity is low, the number of liquidity-shocked bond sellers revealing their liquidity status will eventually reach the maximum possible level as credit risk rises. Then, the size of informed bond selling will increase as credit risk rises further, forming the hump-and-rebound pattern, because the amount of liquidity-shocked sellers whom information-driven sellers can mimic no longer declines from that point.

We show that the hump-and-rebound pattern is consistent with the US corporate bond data. Specifically, we demonstrate that monthly weighted-average yield spreads and turnover rates of corporate bonds indeed exhibit such a hump-and-rebound relationship. We then calibrate the model to quantify the effect of information asymmetry on corporate bond prices.

In calibration, we match the empirically observed turnover rates across different yield-spread groups as closely as possible because the hump-and-rebound relationship between these two variables is one of the unique predictions of our model. Our model performs well in reproducing these cross-sectional empirical moments.

According to the calibrated model, the effects of information asymmetry on corporate bond prices are non-negligible for investment-grade bonds and are sizable for speculative-grade bonds. Specifically, we measure the effect of information asymmetry by the size of informational liquidity costs, defined as the scaled difference between the bond price of anonymous trades in the benchmark model and the bond price in an alternative model without information asymmetry. We show that the price discount attributable to informational illiquidity from 0.51% to 1.14% and that the price discount caused by non-informational frictions ranges from 4.37% to 4.67% for investment-grade bonds. While our model predicts that most investment-grade bonds do not directly suffer from adverse selection in the secondary market, the equilibrium prices of these low-risk bonds still contain positive informational discounts due to the expectation of potential adverse selection in the future.

The size of informational liquidity costs becomes much larger when the bond enters the speculative-grade region. For speculative-grade bonds, the price discount attributable to informational illiquidity from 2.11% to 3.62%, while the price discount caused by non-informational frictions ranges from 5.42% to 6.85%. For these speculative-grade bonds, the size of informational liquidity costs can be 28.0%-34.6% of the size of overall liquidity costs, suggesting severe adverse selection in the secondary bond market.

Furthermore, we conduct comparative statics analysis and provide several policy implications. We find that an increase in the size of non-informational trading costs would increase the yields of most bonds except for bonds with very high credit risks. Thus, any policy that affects transaction costs in the bond market, such as the Volcker rule, may have differentiated effects on bonds with different credit risks due to the policy's effect on informational liquidity in the market. Further, our model suggests that an improvement in accounting transparency or credit-rating informativeness can reduce yield spread by lowering informational liquidity costs, but its effect on trading volume is ambiguous and varies across bonds with different credit risks.

This paper contributes to the quantitative credit-risk literature by developing a structural credit-risk model with information asymmetry in a secondary bond market. In the literature, Ericsson and Renault (2006), He and Xiong (2012), Huang and Huang (2012), He and Milbradt (2014), and Chen et al. (2018) develop structural models to study the impact of liquidity on bond prices. In particular, Ericsson and Renault (2006) investigates the effects of liquidity on the negotiation outcome between equityholders and bondholders. He and Xiong (2012) study the feedback effect between default risk and liquidity risk through a short-term debt rollover channel. He and Milbradt (2014) endogenizes the liquidity risk in He and Xiong (2012), using search frictions. Chen et al. (2018) further extend this model by considering the effects of the business cycle. Our paper contributes to this literature by developing a structural model that enables us to quantitatively examine the effects of information asymmetry among bond investors on corporate bond prices. In the literature, Duffie and Lando (2001) study the effects of informational frictions on the term structures of yield spreads. Their paper assumes that all bond investors possess the same information, while ours focuses on information asymmetry between bond investors.

In this regard, our paper is also related to the theoretical literature examining the implications of information asymmetry in financial markets. A selective list of seminal contributions in this area includes Grossman and Stiglitz (1980), Kyle (1985), Glosten and Milgrom (1985), Eisfeldt (2004), Daley and Green (2012) Malherbe (2014), Biais et al. (2015), Collin-Dufresne and Fos (2016), Daley and Green (2016), and Albagli et al. (2023), among others. Extending this literature, which mainly focuses on stock markets, we embed an information-based model into a structural credit-risk model in an analytically tractable way to quantify the effects of information asymmetry on corporate bond pricing.

Several papers attempt to estimate the adverse selection component in asset prices using the conceptual frameworks developed by the above papers. For instance, Glosten and Harris (1988) presents the empirical evidence on the existence of adverse selection in stock markets by decomposing the bid-ask spread into an adverse-selection component and a transitory component. Stoll (1989) finds that the quoted bid-ask spread of stocks contains a statistically significant information-asymmetry component, whereas George et al. (1991) and Huang and Stoll (1997) show that the economic effect of information asymmetry is smaller, albeit significant, than that of order-processing costs. Our paper extends the strand of this literature by quantifying the effect of information asymmetry in corporate bond markets. In this regard, our paper is close to Han and Zhou (2014), which estimates the contribution of asymmetric information to corporate bond yields using reduced-form models. However, the structural approach in our paper allows us to better capture the equilibrium relation between informed trading and bond pricing. For instance, the methodology in their paper takes the trade size as an exogenous variable when modeling the relationship between the trade size and the informational liquidity component in bond prices. Yet, these two variables are simultaneously determined in our model and affected by common factors such as firm fundamentals, trading costs, and investors' liquidity needs.

Besides these papers backing out information asymmetry from bid-ask spreads, some other papers show evidence of information asymmetry from price movement, trading volume, and other measures of trading activities in stock and option markets; see, for instance, Hasbrouck (1988), Hasbrouck (1991), Lin et al. (1995), Chan et al. (2002), Collin-Dufresne and Fos (2015), and Kacperczyk and Pagnotta (2019). We show that informational frictions can explain the empirically observed non-monotonic relationship between trading volume and yield spread in the bond market. So, our paper suggests that trading volume may not be a good measure of liquidity in markets that suffer from information frictions.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 analyzes the model. Section 4 presents the quantitative results. Section 5 conducts the comparative statics analysis and discusses policy implications. Section 6 presents an alternative model. Section 7 provides the concluding remarks.

2 Model

We develop a structural credit-risk model that incorporates information asymmetry in the secondary bond market into the canonical model by Leland (1994). Time is continuous and runs from 0 to ∞. All agents in the model are risk-neutral and discount future cash flows at a constant risk-free rate of r.

2.1 Firm Assets

Consider a firm with assets that produce a stochastic after-tax cash flow $x_t dt$ over each instantaneous time interval $[t, t+dt)$. The cash flow x_t evolves according to

$$\frac{dx_t}{x_t} = \mu dt + \sigma dZ_t,$$

where μ is a growth rate, $\sigma > 0$ is a volatility, and Z_t is a standard Brownian motion. The first-best value of the assets at time t is then given by the following Gordon's formula:

$$V(x_t) = E_t \left[\int_t^\infty e^{-r(s-t)} x_s ds \right] = \frac{x_t}{r-\mu}. \tag{1}$$

To ensure that the assets have a finite first-best value, we assume $r - \mu > 0$. The realized cash flow x_t is publicly observable. We also call x_t the firm's fundamental at time t.

2.2 Defaultable Bonds

The firm's capital structure is exogenously given. Specifically, the firm has issued a unit mass of perpetual bonds that are diversely held by bondholders. We also assume that any bond investor can hold either 0 or 1 unit of bond, as is commonly assumed in the literature; see, for instance, Duffie et al. (2005), He and Xiong (2012), and He and Milbradt (2014).

Each bond claim pays a constant coupon of c per unit of time. The corporate tax rate is $\tau \in (0, 1)$. Coupon payment is tax deductible, meaning that the net cash flow to equityholders equals $x_t - (1-\tau)c$ at each time t. The net cash flow to equity can be negative, in which case, equityholders need to inject additional capital to cover temporary losses to avoid default. When additional capital injection is deemed unprofitable, however, equityholders decide to default. We postulate that default occurs when the firm's cash flow hits a threshold x_D from above, which is endogenously determined.

When the firm defaults, equityholders hand over the firm's assets to bondholders. The bondholders then immediately liquidate the assets. From liquidation, bondholders recover only a fraction of the first-best value of the assets due to bankruptcy costs. The key feature of the model is that the recovery rate of a firm depends on its intrinsic type, which can be either high (H) or low (L). Denote the recovery rate of high-type firms and low-type firms as α_H and α_L, respectively, where $0 \leq \alpha_L < \alpha_H < 1$. That is, high-type firms incur lower bankruptcy costs than low-type firms.

Under this assumption, the recovery value of a k-type firm will be $\frac{\alpha_k x_D}{r-\mu}$ in default for each type $k \in \{H, L\}$ due to the expression in (1). The probability that a firm is of high type is $\pi \in (0, 1)$. We can also equivalently say that there is a continuum of firms and the fraction of high-type firms among all firms is π. For clarification, each firm's type is predetermined at date 0 and does not change over time.

2.3 Information Structure

In this model, bondholders can sell their bond holdings for various reasons, which will be described below, in the secondary bond market. A common assumption in the existing structural credit-risk models is that bond sellers and buyers in the secondary bond market have the same information about firms. But this assumption is inconsistent with the empirical findings of Han and Zhou (2014) and Benmelech and Bergman (2018), who show that secondary bond markets suffer from information asymmetry. To incorporate this empirical fact in a tractable way, we assume that current bondholders (potential bond sellers) can observe their firm's type precisely, while new bond investors (potential bond buyers) are not informed about the firm's type. For clarification, this assumption means that each new bond investor becomes informed right after purchasing bonds in the market. This assumption is also made in Daley and Green (2016).

In practice, information asymmetry may also arise around some other factors beyond the recovery rate, such as the growth rate or volatility of assets, which generally affect the firm's default probability. However, these factors can be potentially learned from a firm's realized cash flows through financial statements and therefore, the degree of information asymmetry regarding those factors are likely to decline over time. Moreover, even if we assume that bond investors face severe information asymmetry around the factors affecting default probability, the qualitative properties of such a model would be similar to those of our current setup, because information asymmetry in bond markets generally matters when credit risk is high regardless of whether information asymmetry stems from default probabilities or recovery rates, due to the information insensitiveness of superior bonds (Dang et al., 2020). In this regard, for tractability, we parsimoniously assume that information asymmetry arises only around the recovery rate.

2.4 Secondary Bond Market

We now describe bond trading in the secondary bond market. In this model, bondholders sell their bonds either to meet liquidity needs or to exploit their private information about firms. We will see that the second motive for bond selling causes a liquidity discount in bond prices. We also assume that bondholders have a technology that can mitigate such a liquidity problem arising from information asymmetry.

Liquidity-driven Selling: Each bondholder is subject to an uninsurable idiosyncratic liquidity shock that arrives with Poisson intensity $\xi > 0$. Similar to Eisfeldt (2004), Bolton et al. (2011), and He and Xiong (2012), when hit by a liquidity shock, the affected bondholder will face substantially large bond-holding costs and thus decides to sell her bond holdings immediately. Moreover, the liquidity status of a bond seller is not publicly observable, as in Eisfeldt (2004) and Bolton et al. (2011). That is, potential bond buyers cannot directly observe not only a firm's type but also the liquidity status of sellers.

Informed Selling: In the presence of liquidity-shocked bond sellers, non-liquidity-shocked bondholders of low-type firms may have incentives to sell their bonds by mimicking the liquidity-shocked bond sellers from high-type firms, while non-liquidity-shocked bondholders of high-type firms would not have such incentives. This phenomenon is commonly called adverse selection. Accordingly, we postulate that each non-liquidity-shocked bondholder of low-type firms sells her bond holdings with an infinitesimally small probability $m(x_t)dt$ at each time t, where the informed bond-selling strategy $m(x) \in [0, \infty)$ is endogenously determined. Here, bondholders sell their bonds with an infinitesimally small probability because only such an outcome can be sustained in equilibrium, as the measure of liquidity-shocked bondholders is also of dt order.

Throughout, we assume that the bond trading volume of any individual firm is not publicly observable. This assumption, which is also adopted in Grossman and Stiglitz (1980), Bolton et al. (2011), Malherbe (2014), and Zou (2023), rules out the possibility that new bond investors can infer the firm's type from its trading volume. This assumption is acceptable because the trading volume of corporate bonds, which are mostly traded in over-the-counter markets, are not instantly disseminated in the real world. Specifically, even after Trade Reporting and Compliance Engine (TRACE) had been introduced, the information about trading activities, such as uncapped trading volume, is still publicized with several months of delay.[1]

Liquidity Status Revealing: We assume that bond sellers have a technology to mitigate the liquidity problem caused by information asymmetry. This assumption will play a crucial role in obtaining better calibration results.

In principle, we can consider two kinds of technologies that can be used to mitigate information asymmetry. The first is a technology that enables each bond seller to credibly disclose the firm's type, while the second is a technology that allows each bond seller to reveal her liquidity status truthfully. The first technology directly eliminates information asymmetry about the firm's type. The second technology also mitigates information asymmetry to some extent because when a bond seller reveals her liquidity status, potential bond buyers can identify the true trading motive of this bond seller.

While we can potentially incorporate both technologies into our model, we consider only the liquidity-status revealing technology in the main model because this specification allows us to solve the model in a more tractable manner without making any additional simplifying assumptions. We will discuss the rationale underlying this assumption in more detail in Section 2.6. For completeness, in Section 6, we also consider the setting with the first kind of technology, say, direct information disclosure, and demonstrate that the two technologies lead to qualitatively similar outcomes.

Regarding the liquidity-status revealing option, we specifically assume that each bond seller can credibly reveal her liquidity status with a probability $\theta \in [0, \bar{\theta}]$ at costs of $\frac{\delta\theta^2}{2}$. Here, δ measures the size of liquidity-status revealing costs, θ represents the effort level exerted

[1] Financial Industry Regulatory Authority (FINRA) makes historical data about uncapped trading volume available 6 months after the transaction date for corporate and agency transactions and 18 months for securitized product transactions.

to disclose the liquidity status, and $\bar{\theta} \in (0, 1)$ denotes the upper limit on such an effort level. Introducing the upper limit $\bar{\theta}$ on the liquidity-status revealing effort level makes sense because a scenario where all liquidity-shocked bond sellers reveal their liquidity status at the same time is not realistic. As such, this parameter helps us obtain better calibration results. Throughout, the parameter δ will be broadly called the adverse-selection mitigation costs because the qualitative properties of the model will be similar regardless of whether we consider the liquidity-status revealing option or the direct information-disclosure option, as mentioned before.

In addition, note that the choice variable θ depends only on the firm's current fundamental x_t, not on its type, because disclosing the liquidity status is equally costly for all bondholders regardless of their firms' types. Accordingly, we denote the choice variable θ as $\theta(x_t)$, which is hereafter called the liquidity-status revealing strategy, to explicitly indicate that this strategy depends only on the firm's current fundamental, not on its type.

For clarification, notice that non-liquidity-shocked bondholders certainly do not have any incentives to reveal their liquidity status. To formally justify this behavior, we can say that, as off-equilibrium beliefs, if a non-liquidity-shocked bondholder reveals her liquidity status, she would be believed to be a bondholder from a low-type firm and therefore, none of the non-liquidity-shocked bondholders would reveal their liquidity status at costs.

Non-informational Trading Costs: Besides the asymmetric information problem, bond sellers face non-informational frictions, which we model in reduced form as in He and Xiong (2012). Specifically, we assume that each bond seller has to bear additional trading costs of $\kappa(x_t)$ on top of the liquidity discount caused by information asymmetry when selling her bond holdings. These additional trading costs can be broadly regarded as costs caused by search frictions, the monopoly power of dealers, inventory management, and so on. While all qualitative results of our model remain unchanged as long as $\kappa(x)$ is assumed to be weakly decreasing in x, we simply set $\kappa(x)$ to a constant κ, since this specification enables us to solve the model in closed form.

Introducing non-informational trading costs is essential for obtaining realistic outcomes. Specifically, without this additional impediment, all non-liquidity-shocked bondholders of low-type firms would unconditionally sell their bond holdings for the informational motive, which is not realistic. However, in the presence of non-informational trading costs, we will see that the incentives for non-liquidity-shocked bondholders of low-type firms to sell their bonds by exploiting their informational advantages increases as default risk rises, resulting in patterns regarding trading volume consistent with data.

Trading Mechanism: In this model, we assume that bonds are traded bilaterally between a seller and a buyer, as most corporate bonds are traded in the over-the-counter market. However, as mentioned before, we do not explicitly model search frictions via delayed meetings between sellers and buyers. Instead, similar to Easley and O'hara (1987), we assume that each seller can instantly find a buyer at a cost, which is considered to be included in the aforementioned non-informational trading costs, κ.

Once a seller meets a buyer, the seller first decides whether to reveal her liquidity status. After this decision is made, the bond price is determined, depending on whether the seller has revealed her liquidity status or not. Let $P^R(x_t)$ denote the bond price that is set in the case where the seller discloses her liquidity status and let $P^A(x_t)$ represent the bond price that emerges in the case where the seller does not reveal her liquidity status. That is, $P^R(x)$ is the bond price that arises under more transparent trading, while $P^A(x)$ denotes the bond price determined under less transparent trading. We further assume that sellers have all bargaining power and buyers have an outside option with a value of 0. Then, bonds will be priced to ensure that buyers break even.

2.5 Timing

The timing of the events over each instantaneous time interval $[t, t + dt)$ is as follows: (i) the firm's cash flow x_t is realized, (ii) the firm's equityholders make a default decision, (iii) if the firm stays solvent, idiosyncratic liquidity shocks may hit bondholders, (iv) bondholders seeking to sell their bond holdings for the liquidity needs or the informational motive meet with bond buyers, (v) bond sellers decide whether to reveal their liquidity status, and (vi) bond prices are determined to ensure bond buyers to break even, depending on whether bond sellers have revealed their liquidity status.

2.6 Further Discussion of Assumptions

Regarding the assumption on the information structure of this model, the assumption that existing bondholders are better informed than new bond investors is reasonable because of the institutional feature of corporate bond markets, as pointed out by Han and Zhou (2014). Specifically, institutional bondholders generally have the right to attend regular meetings organized by the firm's management and thus can acquire crucial information about the firm. Although we can incorporate informed buyers into our model, we do not pursue this extension because the size of upward price pressure caused by informed buyers is deemed relatively smaller compared to that of downward price pressure caused by informed sellers, as information asymmetry in corporate bond markets tends to push down bond prices on average according to Han and Zhou (2014).[2]

In addition, note that whether equityholders are informed about their firm's type is irrelevant in this model because equityholders, who are supposed to be wiped out in default, are not concerned about the asset recovery rate when making the default decision. As such, in this model, we do not explicitly assume whether equityholders are informed of their firm's type, although assuming equityholders as well as bondholders have access to the information about their firms is more natural.

Next, regarding the liquidity-status revealing option, we can consider situations where institutional investors reveal their liquidity status by showing their balance sheets or their inside information about the recent changes in their fund flows. Once the liquidity status is revealed, bond sellers can avoid informational liquidity discounts because potential bond buyers can detect the true trading motives of the bond sellers who have revealed their liquidity status. As a real-world example, according to Da et al. (2011), many institutional investors looking to offload small-cap stocks for non-informational reasons often approached funds managed by Dimensional Fund Advisors and were able to sold their stocks without facing significant informational liquidity discounts by credibly revealing their true trading motive,

[2]In the previous version of this paper, we develop an extended model in which informed buyers preemptively approach bond sellers and give take-it-or-leave-it offers to those bond sellers. However, as long as we assume that the measure of informed buyers is less than the measure of liquidity-shocked bond sellers, this extension yields qualitatively similar outcomes to our main model. Moreover, this extended model does not substantially improve the quantitative performance of our model. In this regard, we do not include this extended model in the present paper.

which supports our assumption. Moreover, our assumption is conceptually similarly to the assumption adopted by Lee and Wang (2024), who develops a model in which a seller's informedness status is publicly observable with some noise, because both assumptions allow buyers to detect the seller's true trading motive, at least partially, without directly observing the asset's true quality.

As mentioned above, while we consider only the liquidity-status revealing option in our main model, we present an alternative model, in which bond sellers can directly reveal their firm's type at costs, in Section 6. This extension will show that these two adverse-selection mitigation technologies result in qualitatively similar outcomes. Additionally, while we model the adverse-selection mitigation technology in reduced form, we can certainly interpret this technology as an indirect signaling device such as partial asset retention (Myers and Majluf, 1984) or underpricing (Welch, 1989).

3 Model Analysis

In this section, we characterize an equilibrium of the model. We first solve the individual problems of equityholders and bondholders. We then pin down the bond prices that ensure bond buyers to break even. Finally, we construct an equilibrium by examining the conditions that must be jointly satisfied by the individually optimal strategies and bond prices. After characterizing an equilibrium, we also discuss the qualitative properties of the model.

3.1 Individual Problems

This section analyzes the individual problems of equityholders and bondholders. We start with the equityholders' problem, who control the default decision.

Default Strategy: Equityholders in our model encounter the same problem as that in Leland (1994) because equityholders in both models are not concerned about the asset recovery rate in default. Thus, the default threshold x_D is given by

$$x_D = \frac{-\eta_2(r-\mu)(1-\tau)c}{(1-\eta_2)r}, \tag{2}$$

where

$$\eta_1, \eta_2 = \frac{-\mu + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2},$$

as already derived in Leland (1994). The constant η_1 will be used later.

Debt Valuation: We now consider the individual problem of bondholders, who maximize the present value of future profits by optimally choosing a liquidity-revealing strategy and a bond-selling strategy. To this aim, we denote the true value of debt of a k-type firm by $D^k(x)$ for each type $k \in \{H, L\}$.

To solve the bondholder's problem, we see that the standard continuous-time method implies that the value functions $D^H(x)$ and $D^L(x)$ satisfy the following Hamilton-Jacobi-Bellman (HJB) equations:

$$rD^H = c + \xi \left[\max_{\theta(x)\in[0,\bar{\theta}]} \underbrace{\{\theta(x)P^R(x) + (1-\theta(x))P^A(x) - \frac{\delta}{2}\theta(x)^2\}}_{\text{liquidity status revealing}} - \kappa - D^H(x) \right] + \mathcal{A}D^H, \quad (3)$$

$$\text{subject to } D^H(x_D) = \frac{\alpha_H x_D}{r-\mu},$$

and

$$rD^L = c + \xi \left[\max_{\theta(x)\in[0,\bar{\theta}]} \underbrace{\{\theta(x)P^R(x) + (1-\theta(x))P^A(x) - \frac{\delta}{2}\theta(x)^2\}}_{\text{liquidity status revealing}} - \kappa - D^L(x) \right] +$$

$$\max_{m(x)\geq 0} \underbrace{m(x)(P^A(x) - \kappa - D^L(x))}_{\text{informed bond selling}} + \mathcal{A}D^L, \quad (4)$$

$$\text{subject to } D^L(x_D) = \frac{\alpha_L x_D}{r-\mu},$$

where $\mathcal{A}U(x)$ denotes $\mu x U_x(x) + \frac{\sigma^2 x^2}{2} U_{xx}(x)$ for any value function U. Note that these two equations are identical to each other except for the term $m(x)(P^A(x) - \kappa - D^L(x))$ in the second equation for debt valuation of low-type firms.

In both equations, the terms on the left-hand side denote the expected return on debt claims. The term $\theta(x)P^R(x) + (1-\theta(x))P^A(x) - \delta\frac{\theta(x)^2}{2}$ inside the bracket in both equations indicates the expected amount of profits earned by a liquidity-shocked bondholder who has an option to choose the liquidity-status revealing effort level $\theta(x)$ before selling her bond. The solution for this problem is given by

$$\theta(x) = \min\left\{\frac{P^R(x) - P^A(x)}{\delta}, \bar{\theta}\right\}, \tag{5}$$

which indicates that a liquidity-shocked bond seller is more willing to reveal her liquidity status if (i) the liquidity discount caused by informed bond selling, that is, $P^R(x) - P^A(x)$, is larger or (ii) the adverse-selection mitigation cost, δ, is lower. The last terms in both equations, that is, $\mathcal{A}D^H$ and $\mathcal{A}D^L$, indicate the expected changes in the debt value due to changes in the firm's fundamental.

The additional term, $m(x)(P^A(x) - \kappa - D^L(x))$, in equation (4) captures the fact that non-liquidity-shocked bondholders of low-type firms can sell their bond holdings to exploit their informational advantages against new bond buyers. Specifically, if such a bondholder sells her bond today, she would earn $P^A(x) - \kappa$, but if she keeps her bond, the continuation value would be $D^L(x)$. Hence, if this bondholder sells her bond with a probability $m(x)dt$, the net increment in her expected utility would be $m(x)(P^A(x) - \kappa - D^L(x))dt$.

In equilibrium, only two cases can arise: (i) $P^A(x) - \kappa = D^L(x)$ or (ii) $P^A(x) - \kappa < D^L(x)$. That is, the case of $P^A(x) - \kappa > D^L(x)$ can never arise in equilibrium because if this case occurs, all non-liquidity-shocked bondholders of low-type firms would sell their bond holdings, which would then push down the bond price $P^A(x)$ to $D^L(x)$, which in turn eliminates all incentives for selling bonds for the informational motive due to the presence of the additional trading costs of κ, a contradiction.

Accordingly, we can pin down the optimal informed bond-selling strategy as follows:

$$\begin{cases} m(x) \in [0, \infty), & \text{if } P^A(x) - \kappa = D^L(x) \\ m(x) = 0, & \text{if } P^A(x) - \kappa < D^L(x). \end{cases} \tag{6}$$

The first line indicates the case where a non-liquidity-shocked bondholder of a low-type firm is indifferent between selling and keeping her bond, so that $m(x)$ can be any number. The second line indicates the case where such a bondholder strictly prefers to keep her bond.

3.2 Bond Pricing

We now pin down the market price of bonds that ensures bond buyers to break even. Recall that the bond price is determined, depending on whether a bond seller discloses her liquidity status or not. In the first case, that is, when the bond seller discloses her liquidity status, the bond price should be equal to

$$P^R(x) = \bar{D}(x) := \pi D^H(x) + (1-\pi)D^L(x), \tag{7}$$

which is the unconditional expected value of debt. This result obtains because all liquidity-shocked bondholders of both high-type and low-type firms reveal their liquidity status with the same probability $\theta(x)$ and non-liquidity-shocked bondholders never reveal their liquidity status.

In the second case, that is, when a bond seller does not reveal her liquidity status, the bond price is given by

$$P^A(x) = \frac{\pi\xi(1-\theta(x))D^H(x) + (1-\pi)(\xi(1-\theta(x)) + m(x))D^L(x)}{\pi\xi(1-\theta(x)) + (1-\pi)(\xi(1-\theta(x)) + m(x))}, \tag{8}$$

because the total measure of liquidity-shocked bondholders from high-type firms who sell their bonds without revealing their liquidity status is $\pi\xi(1-\theta(x))$, while the total measure of bondholders from low-type firms selling their bonds without disclosing their liquidity status is $(1-\pi)(\xi(1-\theta(x)) + m(x))$. More specifically, the term $(1-\pi)m(x)$ in the second term indicates the total measure of bondholders from low-type firms who sell their bonds for the informational motive. The expressions in (7) and (8) clearly tell that the price $P^A(x)$, which emerges in less transparent trades, is lower than the price $P^R(x)$, which arises in more transparent trades, because of adverse selection.

3.3 Equilibrium

In this section, we construct an equilibrium by using the individual optimality and market-clearing conditions derived above.

3.3.1 Definition

Formally, an equilibrium of this model is defined as a collection of $\{x_D, \theta(x), m(x), P^R(x), P^A(x), D^H(x), D^L(x)\}$ such that (i) the default threshold is given by (2), (ii) the liquidity-status revealing strategy $\theta(x)$ satisfies (5), (iii) the informed bond-selling strategy $m(x)$ satisfies (6), (iv) the bond price that emerges in more transparent trades, $P^R(x)$, satisfies (7), (v) the bond price that arises in less transparent trades, $P^A(x)$, satisfies (8), (vi) the true value of debt of high-type firms, $D^H(x)$, satisfies (3), and (vii) the true value of debt of low-type firms, $D^L(x)$, satisfies (4).

3.3.2 Equilibrium Construction

To pin down an equilibrium, we first postulate that there is a threshold x_S such that non-liquidity-shocked bondholders of low-type firms sell their bonds with a positive probability for the informational motive when $x < x_S$. That is, we conjecture that

$$\begin{cases} m(x) > 0, & \text{if } x < x_S \\ m(x) = 0, & \text{if } x \geq x_S \end{cases}$$

for some threshold x_S, which is endogenously determined. This postulation is reasonable because the private information about recovery rates becomes more valuable when default risk rises. We will depict a typical shape of $m(x)$ in more detail in Section 3.4, considering various cases. For clarification, we rule out the trivial case, in which $m(x) \equiv 0$ for all $x \geq x_D$, which means informed bond selling never occurs, by imposing the following condition:

$$\kappa < \frac{\pi(\alpha_H - \alpha_L)x_D}{r - \mu}. \tag{9}$$

Under this parameter condition, we will see that the informed bond-selling threshold x_S will be always placed above the default threshold x_D.

Taking the informed bond-selling threshold x_S as given, liquidity-shocked bondholders of any types of firms also reveal their liquidity status with a positive probability when their firm's fundamental is below x_S, because the market starts facing informational illiquidity

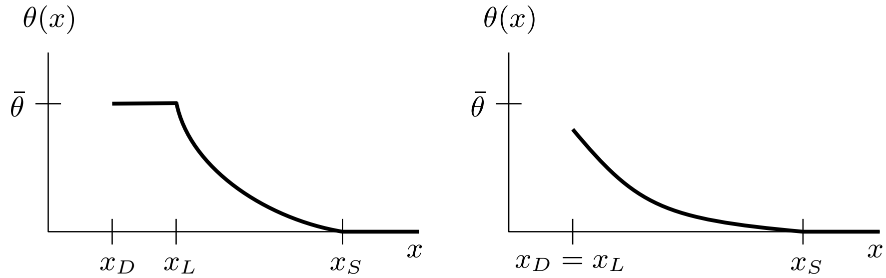

Figure 1: This figure describes a typical shape of the liquidity-status revealing strategy $\theta(x)$. The left panel indicates the case where x_L is placed in the interior region, while the right panel indicates the case where $\theta(x)$ never reaches the maximum possible level.

due to informed bond selling when the firm's fundamental falls below x_S. Moreover, when the firm's fundamental falls further, we can reasonably postulate that the liquidity-status revealing strategy $\theta(x)$ increases because informational illiquidity is deemed to be more severe as default risk rises. We verify this property in Proposition 3.1 when formally characterizing an equilibrium. Hence, our model may have another threshold x_L such that (i) $x_L < x_S$ and (ii) $\theta(x) = \bar{\theta}$ for all x such that $x \leq x_L$.

More specifically, we conjecture that the liquidity-status revealing strategy should be given by

$$\begin{cases} \theta(x) = \bar{\theta}, & \text{if } x_D < x \leq x_L \\ \theta(x) \in (0, \bar{\theta}), & \text{if } x_L < x < x_S \\ \theta(x) = 0, & \text{if } x \geq x_S \end{cases}$$

for some threshold x_L, which is endogenously determined. Of course, such a threshold may not exist because $\theta(x)$ does not necessarily reach the maximum possible point $\bar{\theta}$ even when the firm is about to default. In this case, we just set x_L to x_D for notational convenience. A typical shape of $\theta(x)$ is shown in Figure 1, considering the interior case of $x_D < x_L$ and the corner case of $x_L = x_D$ separately.

To find the above-mentioned two equilibrium thresholds, (x_S, x_L), notice that the bond price arising under less transparent trading, $P^A(x)$, must satisfy

$$\begin{cases} P^A(x) = D^L(x) + \kappa, & \text{if } x_D < x < x_S \\ P^A(x) = \bar{D}(x), & \text{if } x \geq x_S \end{cases} \tag{10}$$

in equilibrium. Specifically, when $x \geq x_S$, as non-liquidity-shocked bondholders of low-type firms do not sell their bonds for the informational motive, the bond price does not exhibit any informational liquidity discount. However, when $x_D < x < x_S$, the market suffers from informed bond selling. In this case, when deriving the property in (6), we have already seen that $P^A(x) - \kappa$ must be equal to $D^L(x)$ to induce non-liquidity-shocked bondholders of low-type firms to be indifferent between selling and keeping their bonds.

Due to the result in (10), the threshold x_S must be a point satisfying

$$\pi J(x_S) = \kappa, \tag{11}$$

where $J(x) := D^H(x) - D^L(x)$ denotes the difference between the true debt values of high-type and low-type firms. In Proposition 3.1, we show that $J(x)$ is strictly decreasing in x. Then, combining with the the parameter condition in (9) and the fact that $\lim_{x\to\infty} J(x) = 0$, we see that there is a unique threshold x_S satisfying condition (11). The fact that $J(x)$ is decreasing in x is intuitive because when default risk increases, bondholders are more concerned about the future recovery value of their firm assets and therefore, the gap between $D^H(x)$ and $D^L(x)$ gets larger.

Regarding the other threshold x_L, note that $P^R(x) - P^A(x)$ in (5) is equal to $\pi J(x) - \kappa$ for all $x < x_S$ due to the property in (10). Then, using the fact that $J(x)$ is strictly decreasing in x again, we see that the threshold x_L is determined as follows:

$$x_L = \min\left\{x \geq x_D : \pi J(x) \leq \kappa + \delta\bar{\theta}\right\}. \tag{12}$$

That is, when $\kappa + \delta\bar{\theta} < \frac{\pi(\alpha_H - \alpha_L)x_D}{r-\mu}$, x_L is the unique interior point that satisfies the condition $\pi J(x_L) = \kappa + \delta\bar{\theta}$. In the other case, the liquidity-status revealing strategy $\theta(x)$ never reaches the maximum possible point, in which case, x_L is just set to x_D as mentioned before. Moreover, since x_S is the point satisfying $J(x_S) = \kappa$, the other threshold x_L should be less than x_S, as postulated.

We have thus far characterized a unique pair of equilibrium thresholds, (x_L, x_S), using the key equilibrium conditions described in (11) and (12). To complete equilibrium char-

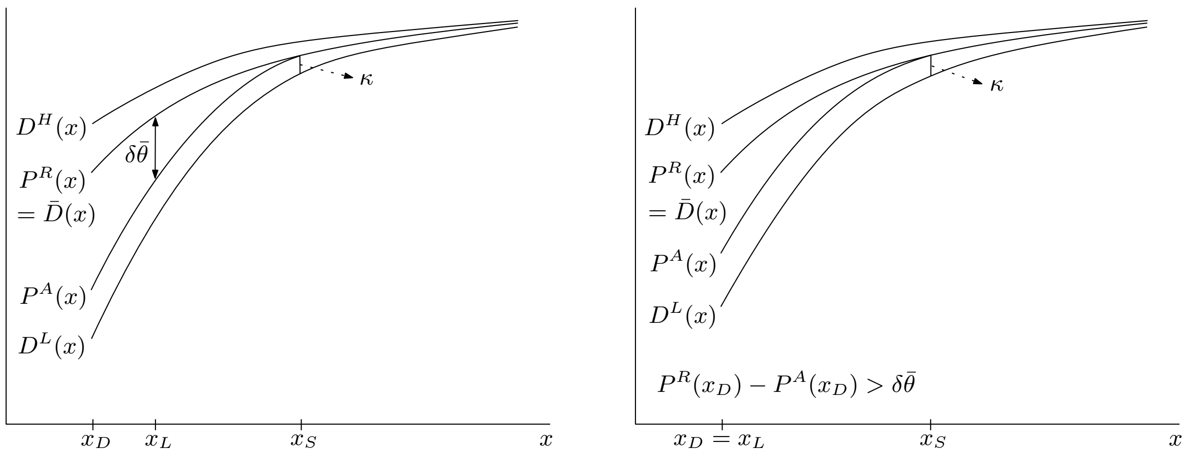

Figure 2: This figure describes the typical shapes of $D^H(x)$, $D^L(x)$, $P^R(x)$, and $P^A(x)$ in equilibrium. The left panel describes the case of $x_D < x_L$ and the right panel depicts the case of $x_L = x_D$, in which the liquidity-status revealing strategy $\theta(x)$ never reaches the maximum possible level.

acterization, we pin down all other equilibrium objects in Proposition 3.1. Moreover, in Section A.2, we provide the closed-form solution for our model for any given pair of thresholds (x_L, x_S) such that $x_L < x_S$. The pair of equilibrium thresholds (x_L, x_S) are computed numerically.

Proposition 3.1. Our model has a unique equilibrium.

Proof. See Appendix A.1. □

Figure 2 depicts the typical shapes of $D^H(x)$, $D^L(x)$, $P^R(x)$, and $P^A(x)$ in equilibrium. The left panel indicates the case of $x_D < x_L$, while the right panel depicts the case of $x_L = x_D$. As shown in the figure, in both cases, $P^A(x) - \kappa$ is less than $D^L(x)$ only when x is above x_S. When x is below x_S, $P^A(x) - \kappa$ is equal to $D^L(x)$. We also see that $P^R(x) - P^A(x)$ is less than $\delta\bar{\theta}$ only when x is above x_L, regardless of whether $x_D < x_L$ or $x_L = x_D$.

3.4 Qualitative Properties

This section discusses the main qualitative properties of our model regarding bond turnover rates and informational illiquidity. Specifically, we first show that the average turnover rate of bonds can exhibit several distinct patterns as default risk increases, depending on cases, in this model. While all those patterns can potentially arise in bond markets, we find that one particular pattern is most consistent with the pattern observed in data, demonstrating that our model can be flexibly calibrated to data. Meanwhile, we show that the informational illiquidity, measured by the price discount caused by informed bond selling, monotonically increases with default risk.

3.4.1 Turnover Rates

To understand the relationship between default risk and turnover rates, we need to study how the informed bond-selling strategy $m(x)$ changes when default risk escalates, because the average bond turnover rate of firms is given by $\xi + \pi m(x)$ in our model. Figure 3 shows the typical shape of the informed bond-selling strategy in three possible cases separately.

Note first that, in all possible cases, the size of informed bond selling, $m(x)$, remains zero when the firm's fundamental lies above the threshold x_S because, by definition, the benefit of exploiting the informational advantages is outweighed by the additional trading costs κ when the fundamental lies in this region. As such, the turnover rate also exhibits a flat pattern when the firm's fundamental stays above x_S. Different patterns in turnover rates emerges when the firm's fundamental falls below x_S.

The left panel shows the special case where δ is sufficiently large. In this case, the graph shows that the informed-bond selling strategy $m(x)$ increases as default risk rises, especially in the region where the firm's fundamental is below x_S. To understand this result, consider an extreme case where δ is set to ∞, which means that bond sellers do not have an option to reveal their liquidity status. In this case, none of the liquidity-shocked bond sellers reveal their liquidity status and thus, the size of the pool of liquidity-shocked bond sellers, whom non-liquidity-shocked bondholders of low-type firms aim to mimic, always stays constant. When this pool size remains constant, non-liquidity-shocked bondholders of low-type firms sell their bond holdings more aggressively to exploit their informational advantages as default risk rises, because private information about recovery rates becomes more valuable as the chance of default increases. As such, $m(x)$ exhibits an increasing pattern when default risk rises, as shown in the first panel of the figure. We can easily see that a similar result obtains when δ is sufficiently large.

The other two panels of Figure 3 illustrate the general case where δ is not sufficiently

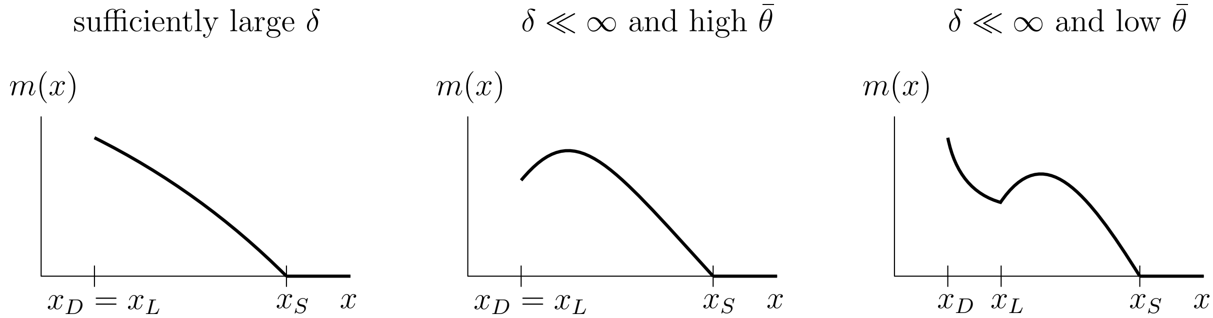

Figure 3: This figure depicts the typical shape of the informed bond-selling strategy $m(x)$ in three possible cases separately. The left panel shows the case where δ is sufficiently large. The middle panel shows the case where δ is moderate or small and $\bar{\theta}$ is high. The right panel shows the case where δ is moderate or small and $\bar{\theta}$ is low.

large, so that liquidity-shocked bond sellers may want to reveal their liquidity status quite aggressively. To analyze this case, we need to further refine the case, depending on whether $\bar{\theta}$ is high or low. Note first that, in both cases, the informed bond-selling strategy $m(x)$ initially increases when the firm's fundamental x falls below the threshold x_S, but starts declining as default risk rises further. However, when default risk surges even further, $m(x)$ keeps declining when $\bar{\theta}$ is high, whereas $m(x)$ rebounds when $\bar{\theta}$ is low.

The initial increasing region arises because private information about recovery rates becomes more relevant as default risk rises, as in the case where δ is sufficiently large. But when default risk increases further, liquidity-shocked bondholders will reveal their liquidity status more aggressively because the informational liquidity discount, $\bar{D}(x)-P^A(x)$, increases with default risk, which we will formally verify later. This outcome means that the size of the pool of liquidity-shocked bond sellers, whom non-liquidity-shocked bondholders of low-type firms seek to mimic, diminishes as default risk rises. Hence, the number of those mimicking bondholders should also decline in equilibrium; otherwise, the indifference condition in the first line of (10) would not hold. Accordingly, $m(x)$ decreases with default risk at least when default risk is not substantially large.

When $\bar{\theta}$ is high, $m(x)$ continues to decline as default risk rises, as shown in the middle panel of the figure. This outcome occurs because the liquidity-status revealing effort level, $\theta(x)$, would not reach the maximum possible level even when default is imminent, if the upper limit of $\theta(x)$ is high.

However, when $\bar{\theta}$ is low, the liquidity-status revealing effort level, $\theta(x)$, will be binding when default risk increases significantly, which means that the number of liquidity-shocked bond sellers who reveal their liquidity status no longer decreases even if default risk rises even further. As such, the incentives of non-liquidity-shocked bondholders of low-type firms to sell their bond holdings by utilizing their informational advantages will resume to increase when default risk soars further, leading to a rebounding pattern for $m(x)$ as shown in the right panel of the figure.

For clarification, when $\bar{\theta}$ is set to 1, $m(x)$ falls to 0 when x hits the threshold x_L from above and remains fixed at that level even if x deteriorates further. This outcome occurs because when $\bar{\theta} = 1$, all liquidity-shocked bondholders will reveal their liquidity status when x falls below x_L, meaning that there are no liquidity-shocked bond sellers whom non-liquidity-shocked bondholders can mimic. While this special case can be analyzed without any additional difficulty, we do not pay much attention to this case, as mentioned above. Later, we will also see that this outcome is not consistent with empirical data.

Now, recall that the average bond turnover rate of firms must exhibit the same patterns as the patterns of the informed bond-selling strategy, $m(x)$, because the average bond turnover rate is equal to $\xi + (1 - \pi)m(x)$ in our model. Given this observation, we formally state the property of the relationship between default risk and turnover rates in the following proposition.

Proposition 3.2. The relationship between default risk and the average turnover rate of firms exhibits (i) an increasing pattern if δ is sufficiently large, (ii) a hump-shaped pattern if δ is not sufficiently large and $\bar{\theta}$ is high, or (iii) a hump-and-rebound pattern if δ is not sufficiently large and $\bar{\theta}$ is low.

Note that our model can generate various patterns in the relationship between default risk and turnover rates because we introduce not only information asymmetry but also the adverse-selection mitigation option to the otherwise standard structural credit-risk model. As such, our model enables us to conduct a more sophisticated quantitative analysis by capturing the nontrivial cross-sectional relationship between default risk and turnover rates observed in data as closely as possible.

3.4.2 Informational Illiquidity

Now we examine the effects of default risk on informational illiquidity, measured by the size of the informational liquidity discount, $\bar{D}(x) - P^A(x)$. Specifically, we show that this informational liquidity discount increases as default risk rises, as depicted in Figure 2.

To show this claim, first note that $\bar{D}(x) = P^A(x)$ for all $x \geq x_S$ as shown in the second line of (10). Next, when $x < x_S$, we have $\bar{D}(x) - P^A(x) = \pi J(x) - \kappa$ due to the first property in (10). Then, since we have already mentioned that $J(x)$ decreases in x, we conclude that $\bar{D}(x) - P^A(x)$ increases as default risk rises, when $x < x_S$.

Proposition 3.3. The informational liquidity discount, $\bar{D}(x) - P^A(x)$, is constantly 0 when $x \geq x_S$, but increases as x dereases in the case of $x < x_S$.

The positive relationship between default risk and informational liquidity discount appears inconsistent with our earlier finding that the size of informed bond selling, $m(x)$, does not necessarily increase as default risk rises. We can understand these two seemingly inconsistent results as follows. Recall that when default risk soars, liquidity-shocked bond sellers reveal their liquidity status more aggressively, meaning that the size of the pool of liquidity-shocked bond sellers whom non-liquidity-shocked bondholders of low-type firms aim to mimic is reduced. Then, as indicated by expression (8), a unit increment in informed bond selling has a bigger downward impact on the the bond price associated with less transparent trading, $P^A(x)$. This observation then implies that the informational liquidity discount, $\bar{D}(x) - P^A(x)$, can be larger when default risk rises, even though the size of informed bond selling, $m(x)$, may decrease due to more aggressive liquidity status revealing.

In the classical papers by Kyle (1985) and Easley and O'hara (1987), who study the impact of informed trading in securities markets, a positive relationship between trading volume and informational price impact arises. In the empirical literature, utilizing the conceptual framework developed by these authors, Glosten and Harris (1988), Huang and Stoll (1997), and Han and Zhou (2014) estimate informational liquidity costs in various asset markets, such as stocks and corporate bond markets. However, in the models of Kyle (1985) and Easley and O'hara (1987), noise traders do not have an option to avoid or mitigate adverse selection, such as the option to reveal their liquidity status, unlike in our model. Such an adverse-selection mitigation option adopted in our model enables us to capture a potentially non-monotonic relationship between asset fundamentals and trading volume. In Section 6, for completeness, we show that this non-monotonic relationship continues to hold even if we consider the direct information disclosure option rather than the liquidity-status revealing option.

4 Quantitative Results

In this section, we present the quantitative results of our model. We first collect corporate bond data and present the empirically observed cross-sectional relationship between yield spreads and turnover rates. We then calibrate the model by matching this relationship as closely as possible. Using the calibrated model, we estimate liquidity costs caused by information asymmetry among bond investors and by non-informational frictions separately.

4.1 Data and Summary Statistics

We obtain the monthly trading volume, bond yields, credit ratings, and other characteristics of corporate bonds from the WRDS Bond Return Database, which is based on bond transactions from FINRA's TRACE data and bond characteristics from Mergent FISD. The data period is from July 2002 to September 2022. Following the literature on corporate bond markets, we focus on bonds issued by non-government US issuers and denominated in US dollars. Further, to focus on plain-vanilla corporate bonds that have better fits with the model, we exclude bonds with zero or variable-rate coupon bonds as well as bonds that are credit-enhanced, convertibles, asset-backed, callable, putable, exchangeable, fungible, preferred, tendered, or a part of a unit deal. We only keep the bonds that are traded on at least 2 distinct months in the sample. We exclude observations on and before the issuance month as well as those on and after the earlier of the default month and the maturity month. This gives us 254,335 bond-month observations for 4,530 bonds.

From this dataset, we compute the yield spread and the annualized turnover rate of each bond-month observation as follows. The yield spread of a bond-month observation is the difference between the bond's yield in that month minus the same-month yield of a

Table 1: Cross-Sectional Statistics for the Yield Spreads and Turnover Rates

Yield spread bins	$(-\infty,0.5)$	[0.5,0.75)	[0.75,1)	[1,1.25)	[1.25,1.5)	[1.5,1.75)	[1.75,2)	[2,2.25)
Yield spread (%)	0.27	0.62	0.87	1.12	1.37	1.62	1.86	2.12
Turnover rate	0.79	0.70	0.65	0.63	0.66	0.69	0.72	0.73
Number of Obs.	39667	27029	23652	20447	18588	16537	13651	11127
Yield spread bins	[2.25,2.5)	[2.5,2.75)	[2.75,3)	[3,3.25)	[3.25,3.5)	[3.5,3.75)	[3.75,4)	[4,4.25)
Yield spread (%)	2.37	2.62	2.87	3.12	3.37	3.62	3.88	4.12
Turnover rate	0.69	0.68	0.65	0.71	0.67	0.76	0.71	0.85
Number of Obs.	8983	7595	6558	5674	4931	4569	4028	3536
Yield spread bins	[4.25,4.5)	[4.5,4.75)	[4.75,5)	[5,5.25)	[5.25,5.5)	[5.5,5.75)	[5.75,6)	[6,6.5)
Yield spread (%)	4.38	4.62	4.88	5.12	5.37	5.62	5.87	6.23
Turnover rate	0.80	0.92	0.88	0.87	0.85	0.86	0.89	0.93
Number of Obs.	3051	2606	2286	1884	1668	1459	1327	2087
Yield spread bins	[6.5,7)	[7,7.5)	[7.5,8)	[8,8.5)	[8.5,9)	[9,9.5)	[9.5,10)	$[10,+\infty)$
Yield spread (%)	6.73	7.25	7.75	8.26	8.74	9.24	9.74	13.86
Turnover rate	1.07	0.97	1.27	1.25	1.21	1.31	1.35	1.47
Number of Obs.	1477	1198	1074	807	669	566	407	4633

Notes: The table contains numbers of observations, weighted averages of yield spreads, and weighted averages of annualized turnover rates for bond-month observations in different yield-spread bins.

treasury bond with the same maturity as the corporate bond.[3] If there is no trade in that month, we use the bond's yield spread in the most recent month with active trading. The annualized turnover rate of a bond-month observation is computed by the bond's par-value trading volume in that month, obtained from the WRDS Bond Return Database, multiplied by 12, and divided by the bond's par-value outstanding amounts obtained from the same database. We winsorize yield spreads and turnover rates of the whole sample at the 0.5% level. Throughout the paper, we use S&P ratings to classify corporate bonds.

To use the empirical relationship between yield spreads and turnover rates for calibration, we group all bond-month observations in 31 bins by yield spread. As in Table 1, the bin width is 25 basis points (bps) when the yield spread is below 6% and is 50 bps otherwise. We widen the bin interval because the number of observations is much smaller for bonds with higher yield spreads. For each bin, we compute the weighted average of yield spreads and the weighted average of annualized turnover rates, where we weight each bond-month

[3]The time-series data of treasury yields are obtained from St. Louis's FRED Economic data. Because FRED provides the yields of treasuries with only specific maturities, we use linear interpolation to construct the yield curve.

observation by the bond's par-value outstanding amount in that month to rely more on data from bonds with larger outstanding amounts.

Figure 4 plots the weighted averages of yield spreads and annualized turnover rates for bond-month observations in different yield-spread bins. The relationship between yield spreads and turnover rates in these yield-spread bins is generally consistent with the model's prediction. The turnover rate does not vary much with the yield spread for low-yield-spread bins, defined as those below the [3.75,4) bin. Yet, the turnover rate becomes a non-monotonic function of the yield spread for high-yield-spread bins, those above the [3.75,4) bin. We will discuss more about this non-monotonic relationship in Section 4.3.

Note that in Figure 4, we exclude the lowest-yield-spread bin, which contains observations with yield spreads below 50 bps. The turnover rate of that bin is 0.79, which is slightly higher than the average turnover rate of adjacent bins with higher yield spreads. This result, which our model does not predict, can arise in reality because of the clientele effect (Amihud and Mendelson, 1986). That is, bonds with lower yield spreads are more likely to be held by investors with shorter investment horizons and are thus traded more frequently. We will discuss the clientele effect in more detail in Section 5.2. Besides, we also exclude the highest-yield-spread bin, which contains observations with yield spreads above 10%. This is because the average yield spread of that bin is 13.86%, which falls into the default region in the calibrated model.

4.2 Model Calibration

In this section, we calibrate the model parameters. We set the risk-free rate r to 4% because the average 10-year US Treasury rate from 2002 to 2022 is about 3.76%. Given that the US corporate tax rate is 35%, we set the tax rate π to 27%, taking into account tax exemptions applied to institutional bond investors; see He and Xiong (2012) and He and Milbradt (2014) for the detailed arguments. The average asset payout ratio estimated by Zhang et al. (2009) and Huang et al. (2020) is about 2%. So, we set the asset growth rate μ to 2% because the asset growth rate is equal to the risk-free rate minus the asset payout ratio in the risk-neutral world. We set the asset volatility σ to 20% because the average asset volatility of

Table 2: Baseline Parameter Values

risk-free rate	$r = 4\%$
corporate tax rate	$\tau = 27\%$
asset growth rate	$\mu = 2\%$
asset volatility	$\sigma = 20\%$
coupon payment	$c = 5.73$
liquidity shock intensity	$\xi = 0.68$
recovery rate of high-type firms	$\alpha_H = 62\%$
recovery rate of low-type firms	$\alpha_L = 53\%$
proportion of high-type firms	$\pi = 50\%$
non-informational trading costs	$\kappa = 0.33$
adverse-selection mitigation costs	$\delta = 0.47$
capacity limit of the mitigation technology	$\bar{\theta} = 0.79$

corporate bonds estimated by Zhang et al. (2009) is about 20%. We set the amount of coupon payments, c, to match the average yield spread of BBB-rated bonds. Specifically, according to our data, the average yield spread of BBB-rated bonds is 173 bps. Then, by normalizing the average price of BBB-rated bonds to 100, we set the coupon payment c to 5.73 because the yield spread of perpetual bonds with a price P is given by $\frac{c}{P} - r$. We will later see that the cash-flow level corresponding to BBB-rated bonds is $x_{BBB} = 2.54$ in our model.

Regarding the liquidity-shock intensity ξ, recall that the turnover rate of bond with a fundamental higher than x_S is equal to ξ in our model because $m(x) = 0$ when $x \geq x_S$. In other words, the turnover rate of bonds with low credit risk does not vary in response to changes in credit risk. This property is consistent with data. Specifically, according to Table 1, the turnover rates of bonds belonging to the bins with yield spreads from 50 bps to 300 bps are relatively flat, compared to the bonds in other bins. As such, we can reasonably set ξ to the average turnover rate of those bonds, which is equal to 0.68. As discussed above, we do not include the bins with yield spreads less than 50 bps because the turnover rates of bonds in these bins are considerably higher than those of other investment-grade bonds. Although we can consider the clientele effects introduced by Amihud and Mendelson (1986) to explain this phenomenon, we do not pursue to add this feature to our model mainly because the focus of our paper lies on the behavior of speculative bonds. A recent paper by Chen et al. (2020) conducts intensive analysis to examine the liquidity premium caused by the clientele effect in corporate bond markets.

To choose the firm-specific recovery rates, we utilize the average recovery rates estimated by Glover (2016) for different industries. Specifically, according to Glover (2016), the average recovery rate of firms in industries within the upper half percentile of recovery rates is about 62%, while the average recovery rate of firms in industries within the lower half percentile is around 53%. So, we set α_H to 62% and α_L to 53%. To be consistent with this simple calibration method, we set the proportion of high-type firms, π, to 50%. Such an unbiased number for the proportion of high-type firms is also used in Eisfeldt (2004). Under this parameter choice, the average recovery rate is 57.5%, which is close to the average recovery rate estimated by Alderson and Betker (1995), Chen (2010), and Glover (2016).

We calibrate the remaining parameters, κ, δ, and $\bar{\theta}$, to jointly match empirical moments regarding trading costs and turnover rates as closely as possible. First, we match the empirically observed total trading costs of junk bonds rated BB or B. According to Edwards et al. (2007), the total trading costs for these bonds with the average trade size of \$200,000 are estimated to about 70 bps. Here, we have chosen \$200,000 as the representative trade size because this number is close to the medium trade size for all bond trades, as in He and Milbradt (2014). According to our data, the average yield spread between BB-rated and B-rated bonds is equal to 451 bps. Then, note that, in our model, the average total trading costs are defined as $(\bar{D}(x) - \bar{P}(x) + \kappa)/\bar{D}(x)$ in percentage terms, where $\bar{P}(x)$ indicates the average bond trading price, which is given by $\frac{\xi\theta(x)}{\xi+(1-\pi)m(x)}P^R(x) + \frac{(\xi(1-\theta(x))+(1-\pi)m(x))}{\xi+(1-\pi)m(x)}P^A(x)$. We match this model-implied total trading costs for junk bonds to 70 bps. We will later report the cash-flow level corresponding to this junk bond group.

Second, we match the empirically observed turnover rates of bonds across different yield-spread bins as closely as possible, which are summarized in Table 1. Specifically, in this calibration, we use the bins from 300 bps to 1000 bps because we have already used the bins from 50 bps to 300 bps when estimating the liquidity-shock intensity, ξ, utilizing the property that the turnover rates of bonds with low credit risk are relatively flat. Also, we use the mean squared error to target these cross-sectional moments regarding turnover rates. In the quantitative credit-risk literature, our paper is the first to match the cross-sectional turnover rates, which exhibit a highly nonlinear pattern.

The parameter values calibrated from the above-described approach are $\kappa = 0.33$, $\delta =$

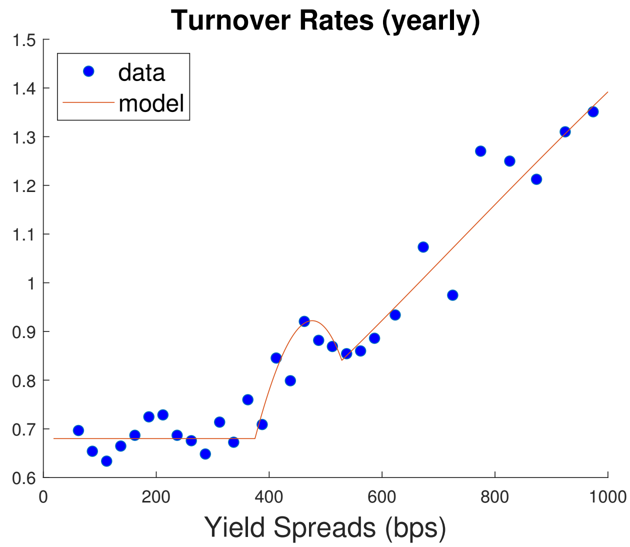

Figure 4: This figure plots both the empirically observed and model-implied turnover rates against the yield spreads of bonds. The cross-sectional empirical moments of the turnover rate are taken from Table 1. For the model-implied result, the baseline parameter values in Table 2 are used.

0.47, and $\bar{\theta} = 0.79$. All the baseline parameter values estimated in this section are summarized in Table 2. Under the baseline parameter values, the equilibrium thresholds x_D, x_L, and x_S are given by 1.23, 1.54, and 1.75, respectively. In addition, the cash-flow levels corresponding to BBB-rated, BB-rated, and B-rated bonds are $x_{BBB} = 2.54$, $x_{BB} = 1.71$, and $x_B = 1.56$, respectively. Note that $x_{BB} < x_S$, indicating that informed bond selling starts to occur slightly before the credit rating of bonds falls to BB. In the subsequent sections, we will discuss the calibration results in more detail and decompose the overall liquidity costs into the informational component and the non-informational component, using our calibrated parameter values.

4.3 Turnover Rates and Yield Spreads

Figure 4 plots model-implied and empirically observed turnover rates across different yield-spread bins. The calibrated model predicts a non-monotonic relationship between yield spreads and turnover rates in the hump-and-rebound pattern, which is consistent with empirical data. We categorize the relationship into three regions and discuss them separately. In the calibrated model, this no-informed-trading region span between 0 and 374 bps, which includes the yield spreads of most investment-grade bonds.

Firstly, when the yield spread is less than 374 bps, the turnover rate in the calibrated model does not vary with a change in the yield spread. In this no-informed-trading region, bonds have low default risks, so non-liquidity-shock bondholders have little incentive to engage in informed trading. As such, the turnover rate is determined by the intensity of liquidity shock arrivals and does not vary with the bond yield.

Secondly, when the yield spread is between 374 and 529 bps, the turnover rate in the calibrated model is a hump-shaped function of the yield spread. In this hump-shaped region, an increase in the yield spread has two opposite effects on trading volume. On the one hand, it makes the bondholder's private information more valuable, inducing more non-liquidity-shocked bondholders to conduct informed selling and thus increasing the overall trading volume. On the other hand, it increases liquidity-shocked investors' incentives to reveal their liquidity status so as to avoid the heightened informational costs in the secondary market. As discussed in Section 3, more liquidity-status revealing reduces the expected profit of informed selling, which tends to decrease the overall trading volume.

In the calibrated model, we find that the former informed-trading effect dominates when the yield spread is between 374 and 477 bps, while the latter information-revealing effect dominates when the yield spread is between 477 and 529 bps. Consequently, the relationship between yield spread and turnover rate is hump shaped in this region, which is driven by the endogenous interaction between non-liquidity-shocked bondholders' informed selling and liquidity-shocked bondholders' liquidity-status revealing. Note that the average yield spreads of BB- and B-rated bonds are in this hump-shaped region, which indicates that informed trading and information revealing start to affect secondary market liquidity when the bond rating enters the speculative-grade region.

Lastly, when the yield spread is higher than 529 bps, the turnover rate becomes strictly increasing in the yield spread. In this region, the bondholders' liquidity-status revealing efforts hit the maximum level $\bar{\theta}$. Then, an increase in yield spread would encourage more informed selling by non-liquidity-shocked bondholders without increasing liquidity-status revealing by liquidity-shocked bondholders, resulting in higher trading volume in the secondary market. In our sample, this informed-selling region consists mostly of high-yield bonds with CCC or lower ratings.

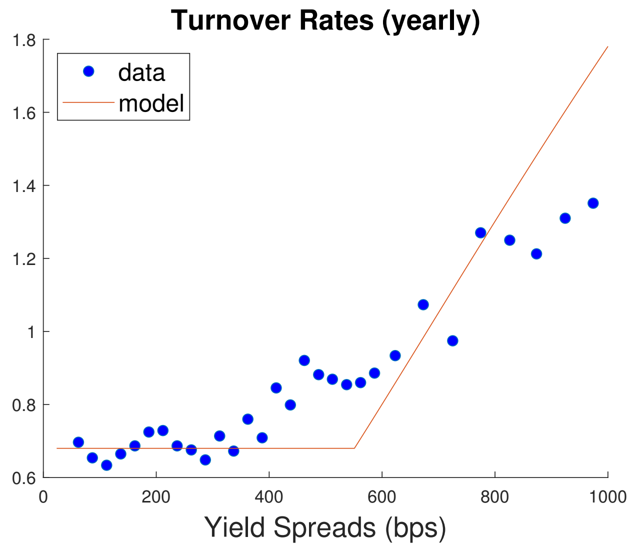

Figure 5: This figure plots the alternative calibration result if we shut down the adverse-selection mitigation channel by setting δ to infinity. For this result, the parameter values of α_H, α_L, and κ are appropriately replaced by 0.60, 0.55, and 0.44, respectively, to better fit the data. All other parameter values are taken from the baseline parameter values in Table 2.

The calibration results raise concerns about using the turnover rate or other volume-based metrics as measures for bond market liquidity. In search-based models such as Duffie et al. (2005) and He and Milbradt (2014), the turnover rate of a bond, which is determined by the searching technology in the bond market, is indeed positively correlated with other price-based-measures of bond market liquidity. However, our model suggests that informational friction complicates the relationship between turnover rate (or other volume-based liquidity measures) and transaction costs (or other price-based liquidity measures). According to our calibration, an increase in the default risk of a bond can cause the turnover rate and the liquidity costs to move in opposite directions, specifically when the bond yield is in the informed-selling region (above 529 bps) or in a part of the hump-shaped region (between 374 and 477 bps), which corresponds mostly to speculative-grade bonds. Therefore, our model suggests that trading volume may not be a good measure of liquidity in markets that suffer from informational frictions.

Our model features the endogenous interaction between informed trading by non-liquidity-shocked bondholders and liquidity-status revealing by liquidity-shocked bondholders. While the presence of informed trading can be easily seen from the empirical fact that high-yield-spread bins have high turnover rates, it is less obvious that the adverse-selection mitigation option, specified as the liquidity-status revealing in the main model, is necessary for matching the empirical relationship between yield spread and turnover rate. To highlight the role of the adverse-selection mitigation option, Figure 5 presents the calibration result of a setting where we shut down the liquidity-status revealing.

In the model with no liquidity-status revealing, Figure 5 shows that the turnover rate is flat in the yield spread for low-yield bonds and is monotonically increasing in the yield spread for high-yield bonds. Nevertheless, the calibrated model with no liquidity-status revealing does not match the empirical relationship between turnover rate and yield spread very well. Compared to the empirical data plotted in blue dots, the turnover-rate increase in the calibrated model happens at a higher yield spread value and is much steeper. Comparing the calibration result in Figure 4 with that in Figure 5, we show that the introduction of a costly technology allowing liquidity-shocked bondholders to mitigate adverse selection in the secondary market is needed to match the empirical relationship between turnover rate and yield spread.

4.4 Decomposition of Liquidity Costs

In this section, we study the quantitative importance of informational and non-informational liquidity costs in the calibrated model. To this end, we first consider a benchmark model developed by Leland (1994), which does not have any liquidity frictions in the secondary bond market. Let $D^B(x)$ denote the debt value obtained in this benchmark model. We then consider another hypothetical model that incorporates only non-informational trading costs into Leland (1994) by assuming that bondholders need to pay κ to sell their bond holdings when hit by liquidity shocks. That is, this model captures only non-informational trading costs, but not informational trading costs. Let $D^N(x)$ denote the debt value obtained in this model. We omit to present the closed-form solutions for this model because the solutions are similarly derived as in He and Xiong (2012).

Then, we first measure the size of non-informational liquidity costs as $\frac{D^B(x)-D^N(x)+\kappa}{D^B(x)}$ for bonds with the current fundamental of x. Here, when we consider the model with only

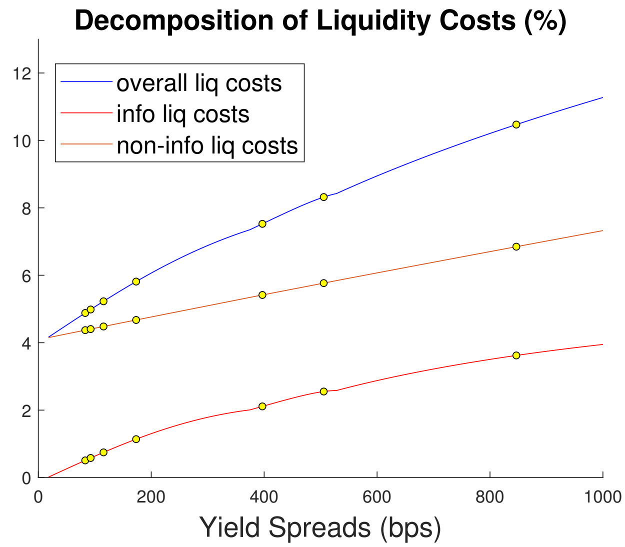

Figure 6: This figure plots how the overall liquidity costs are decomposed into the informational and non-informational parts. The blue curve plots the overall liquidity costs. The red curve plots the informational liquidity costs. The green curve plots the non-informational liquidity costs. The yellow dots on each curve correspond to the average yield spreads for AAA, AA, A, BBB, BB, B, and CCC, respectively, from left to right.

non-informational trading costs, we set the recovery rate to 57.5%, which is the same as the average recovery rate in our main model with both informational and non-informational frictions. However, if we also set the recovery rate in the benchmark model with no liquidity frictions to the same level, the non-informational liquidity costs will converge to 0 as the chance of default increases, as also pointed out by He and Milbradt (2014). The reason is that, in this case, bondholders will no longer face liquidity shocks after default and therefore, the default event would be actually beneficial for bondholders to some extent. As such, He and Milbradt (2014) incorporate liquidity costs in secondary asset markets in reduced form by setting the recovery rate in the benchmark model to be appropriately higher than that in the model with liquidity frictions. To adopt this point in the simple manner, we use the empirical finding of Acharya et al. (2007) that the recovery rate of firms in financially distressed industries is less than that in financially healthy industries by about 8%, if we exclude utilities and financial service industries. Employing this fact, we set the recovery rate in the benchmark model without any liquidity frictions to $57.5 \times 1.08 = 62.1\%$.

Next, we measure the size of overall liquidity costs by $\frac{D^B(x)-\bar{P}(x)+\kappa}{D^B(x)}$ as $\bar{P}(x)$ denotes the average bond trading costs in the model with both informational and non-informational frictions. Then, the informational part of the overall liquidity costs is naturally defined by the difference between overall liquidity costs and non-informational liquidity costs, measured above.

Figure 6 presents the decomposition of overall liquidity costs into informational and non-informational liquidity costs. We first find that the size of informational liquidity costs is non-zero for bonds in the no-informed-trading region, in which the yield spread lies between 0 and 374 bps according to Figure 4. To explain this result, note that the size of informational liquidity costs is measured as the percentage difference between the average bond price in our main model and that in the hypothetical Leland model with non-informational costs only. For a bond whose yield spread is in the no-informed-trading region, although there is no informed selling in the current bond market, its bondholder may suffer adverse selection in selling her bond in the future when its yield spread significantly increases. Due to the dynamic nature of the model, our measure of informational liquidity costs takes into account the present value of future informational liquidity costs, which is non-zero for most high-quality bonds.

Next, we analyze the size of informational liquidity costs, which can be interpreted as the bond-price discount caused by the negative impact of adverse selection on the secondary bond market. As plotted in Figure 6 and reported in Table 3 (the benchmark case), the sizes of informational liquidity costs are 0.51%, 0.58%, 0.75%, and 1.14% for bonds whose yield spreads are equal to the weighted average yield spreads of AAA, AA, A, and BBB rated bonds, respectively. Therefore, for investment-grade bonds with low credit risks, the presence of informational friction generates a positive but relatively small price discount when compared to the frictionless benchmark.

For speculative-grade bonds, the size of informational liquidity costs is much larger. According to the calibration result, the sizes of informational liquidity costs are 2.11%, 2.55%, and 3.62% for bonds whose yield spreads are equal to the weighted average yield spreads of BB, B, and CCC rated bonds, respectively.

In sum, our calibrated model shows that adverse selection negatively affects market liquidity not only for high-risk bonds that are currently subject to informed trading but also for low-risk bonds due to the expectation of future informational illiquidity. In the empirical literature, Benmelech and Bergman (2018) find that the effect of information asymmetry on market liquidity exhibits a hockey-stick pattern and is sizable only for bonds with significant default risks. Our calibration result is consistent with this hockey-stick pattern in the sense that the amount of informed trading is zero for bonds with yield spreads below 374 bps. Nevertheless, we show that when the presence value of future informational liquidity costs is considered, the size of informational liquidity costs corresponds roughly to a price discount of 0.5% to 1% for investment-grade bonds, which is still economically significant.

Before discussing the model's comparative statics results, we compare the relative magnitude of informational and non-informational liquidity costs for bonds with different yield spreads. For investment-grade bonds, our results suggest that their informational liquidity costs take up 10.5%-19.6% of the overall liquidity costs. However, for speculative-grade bonds, we find that their information liquidity costs take up 28.0%-34.6% of the overall liquidity costs. These results show that the effect of informational liquidity costs in determining bond market liquidity is economically significant and is higher for bonds with higher credit risks.

5 Comparative Statics and Policy Implications

5.1 Effects of Non-Informational Trading Costs

We first analyze the effect of the non-informational trading costs on yield spread and informational liquidity. The results are summarized in Table 3, which plots yield spreads, turnover rates, informational and non-informational liquidity costs of four investment-grade representative bonds and four speculative-grade representative bonds for three different levels of non-informational trading costs κ. These representative bonds are chosen so that the yield spread of each representative bond corresponds to the weighted average yield spread for each rating class in the calibrated model.

As shown in Table 3, reducing non-informational trading costs κ weakly increases turnover rates for bonds at all rating classes (from AAA to CCC). For the four investment-grade representative bonds, the turnover rate is not sensitive even for a large decrease of κ from 0.6 to 0.1. In contrast, for the four speculative-grade representative bonds, a decrease in κ

Table 3: The Effects of the Non-Informational Trading Costs

		AAA	AA	A	BBB	BB	B	CCC
$\kappa = 0.1$	Yield spreads	70	80	102	159	385	496	850
	Turnover rate	0.68	0.68	0.68	0.68	1.45	1.45	2.79
	Overall liquidity costs	2.14	2.26	2.55	3.26	5.87	7.03	10.29
	Informational part	0.53	0.60	0.77	1.18	2.64	3.26	4.84
	Non-informational part	1.61	1.66	1.78	2.08	3.23	3.78	5.45
$\kappa = 0.33$	Yield spreads	83	93	115	173	397	506	846
(benchmark)	Turnover rate	0.68	0.68	0.68	0.68	0.77	0.90	1.22
	Overall liquidity costs	4.88	4.99	5.23	5.81	7.53	8.32	10.47
	Informational part	0.51	0.58	0.75	1.14	2.11	2.55	3.62
	Non-informational part	4.37	4.40	4.48	4.67	5.42	5.77	6.85
$\kappa = 0.6$	Yield spreads	99	109	132	191	416	521	851
	Turnover rate	0.68	0.68	0.68	0.68	0.68	0.72	0.91
	Overall liquidity costs	8.13	8.21	8.41	8.87	10.05	10.28	11.30
	Informational part	0.51	0.58	0.75	1.15	2.06	2.17	2.82
	Non-informational part	7.62	7.63	7.65	7.72	7.98	8.11	8.48

Notes: This table shows the effects of the non-informational trading costs κ on yield spreads (bps), turnover rates (yearly), overall liquidity costs (%), informational liquidity costs (%), and non-informational liquidity costs (%) across different rating classes.

significantly increases their trading volume. For instance, when κ decreases from the benchmark case 0.33 to 0.1, the turnover rate increases by 88%, 61%, and 129% for the average BB- , B- and CCC-rated bonds, respectively. The turnover-rate increase is primarily driven by more informed trading on these speculative bonds. Among the three rating classes, the turnover rate increase is smallest for the average B rated bond, which reflects the impact of liquidity-shocked bondholders' liquidity-status revealing activities on suppressing informed selling.

Next, we analyze the impact of κ on bond pricing by looking at its effect on the yield spread. Theoretically, a decrease in the size of non-informational trading costs reduces yield spread by directly decreasing non-informational liquidity costs, yet it may raise yield spread by inducing more informed trading and increasing informational liquidity costs. Therefore, the effect of κ on bond yield spread is qualitatively ambiguous. Quantitatively, our calibrated model shows that the effect of non-informational trading costs on yield spread is heterogeneity across different rating classes. For instance, a decrease of κ from the benchmark case 0.34 to 0.1 in Table 3 reduces yield spread by roughly 13 bps for the average AAA-, AA-, A-, BBB-and BB-rated bonds, by 10 bps for the average B-rated bond, and by -4 bps for the average CCC-rated bond. We see that a decrease in non-informational trading costs leads to lower yield spreads for most bonds. However, for bonds with very high credit risks, a decrease in non-informational trading costs may actually cause an increase in yield spread by worsening the adverse selection problem in the secondary market.

The above comparative-static results concerning non-informational trading costs generate a few interesting policy implications. First, we can interpret the effect of the Volcker rule on the corporate bond market as an increase in the non-informational trading costs in the model. The Volcker Rule, intended to limit bank risk-taking by restricting certain speculative activities, had the unintended consequence of reducing regulated banks' market-making activities, which may cause a decrease in bond market liquidity. For instance, Bao et al. (2018) find empirical evidence that due to the Volcker Rule, bonds have become less liquid during stress when bonds are downgraded to junk status. Consistent with this empirical finding, our model predicts that increasing the trading costs would generally reduce market liquidity and thus increase yield spreads. However, our model predicts that for bonds with very highly speculative credit quality (rated CCC or below), the increase in trading costs may actually reduce bond yields due to its quantitatively large effect on mitigating the adverse selection problem in the bond market.

In addition, the comparative-static results can be used to analyze the effect of electronic trading platforms on corporate bonds. In recent years, the usage of electronic venues in corporate bond trading has become more prevalent. Compared to traditional over-the-counter trading, electronic trading platforms can reduce transaction costs by allowing investors to search many bond dealers simultaneously and to obtain pre-trade information more easily. Hendershott and Madhavan (2015) find that controlling for bond rating and trade size, trading costs are substantially lower in electronic trading platforms than in OTC voice-based trading. According to our model, while a decrease in the trading costs in the model reduces yield spreads for bonds at most rating classes, it also induces more informed trading, especially for speculative-grade bonds. As such, a decrease in the non-informational trading cost makes informational illiquidity more critical in determining bond prices.

Facing a higher degree of adverse selection, liquidity-shocked bond sellers reveal their liquidity status more aggressively to trade at more favorable prices. In this regard, our model predicts that a decrease in trading costs caused by the development of electronic trading platforms increases the demand for liquidity-status revealing. We postulate that liquidity-status revealing is more likely to happen when an investor trades with her relationship dealer, who is better informed of her trading motive but charges a higher price due to information monopoly. Therefore, our comparative-static results suggest a certain level of segmentation in corporate bond markets, where an investor searches in the electronic trading platform when trading low-risk information-insensitive bonds and would turn to her relationship dealer when trading high-risk bonds that may cause concerns about adverse selection.

5.2 Effects of the Intensity of Liquidity Shocks

Table 4 presents the comparative-static results concerning ξ, the Poisson intensity at which a bondholder is hit by a liquidity shock and forced to sell her bond position. First, Table 4 shows that an increase in the liquidity shock intensity leads to increases in turnover rates and non-informational liquidity costs for all rating classes. These two results are not surprising: when bond investors are more likely to liquidate their bonds, the trading volume is higher, and the expected value of potential trading costs due to forced selling in the future becomes larger.

As for informational liquidity costs, our calibration results suggest that an increase in ξ tends to increase the informational liquidity discount in the bond market. When the liquidity-shock intensity increases, there are more liquidity-driven sales in the secondary market. More liquidity-driven selling induces more non-liquidity-shocked bondholders to exploit their informational advantages, resulting in a higher degree of adverse selection and a higher informational liquidity discount.

Last, we analyze the effect of liquidity shock intensity on yield spread in the calibrated model. According to Table 4, an increase in the liquidity shock intensity leads to increases in yield spreads for all rating classes, and the yield-spread increase is larger for high-yield bonds. For instance, when we increase ξ from 0.68 (benchmark case) to 0.9, the yield-spread increase is about 7 bps for investment-grade bonds, and increases to about 10 bps for speculative-grade

Table 4: The Effects of Liquidity-Shock Intensity

		AAA	AA	A	BBB	BB	B	CCC
$\xi = 0.5$	Yield spreads	78	88	110	167	390	498	839
	Turnover rate	0.50	0.50	0.50	0.50	0.66	0.64	0.94
	Overall liquidity costs	3.89	4.00	4.25	4.85	6.70	7.50	9.95
	Informational part	0.50	0.56	0.72	1.08	2.00	2.36	3.46
	Non-informational part	3.39	3.43	3.53	3.77	4.70	5.14	6.49
$\xi = 0.68$	Yield spreads	83	93	115	173	397	505	846
(benchmark)	Turnover rate	0.68	0.68	0.68	0.68	0.78	0.89	1.22
	Overall liquidity costs	4.88	4.99	5.23	5.81	7.53	8.32	10.47
	Informational part	0.51	0.58	0.75	1.14	2.11	2.55	3.62
	Non-informational part	4.37	4.40	4.48	4.67	5.42	5.77	6.85
$\xi = 0.9$	Yield spreads	89	99	122	180	406	515	855
	Turnover rate	0.90	0.90	0.90	0.90	0.90	1.21	1.55
	Overall liquidity costs	6.09	6.18	6.41	6.96	8.57	9.26	11.07
	Informational part	0.52	0.59	0.76	1.18	2.28	2.72	3.79
	Non-informational part	5.57	5.59	5.64	5.78	6.29	6.54	7.29

Notes: This table shows the effects of the liquidity-shock intensity ξ on yield spreads (bps), turnover rates (yearly), overall liquidity costs (%), informational liquidity costs (%), and non-informational liquidity costs (%) across different rating classes.

bonds.

The liquidity shock in the model corresponds to bond selling for non-informational reasons such as preference changes, leverage constraints, and forced redemptions in bond ETFs or mutual funds. According to the calibrated model, shocks that force bond investors to liquidate more often, i.e., performance-driven mutual fund redemptions (Goldstein et al., 2017), would have larger adverse effects on bond prices for high-yield bonds. Nevertheless, expecting higher costs associated with liquidating these high-yield bonds, a bond mutual fund manager with a diverse portfolio of corporate bonds may choose to sell high-quality corporate bonds to minimize liquidation costs (Ma et al., 2022). Incorporating bond investors' endogenous liquidation decisions into our credit-risk model, which is beyond the scope of our paper, can be valuable for understanding the interaction between investors' liquidation decisions and equilibrium bond pricing.

Another factor that affects the liquidity-shock intensity in the model is the liquidity preference of bond investors. For instance, Chen et al. (2020) tests the clientele effect in the corporate bond market and finds that insurers' investment horizons and funding constraints

Table 5: The Effects of the Recovery Rate Difference

		AAA	AA	A	BBB	BB	B	CCC
$\alpha_H = 59\%$	Yield spreads	81	91	113	169	385	488	807
$\alpha_L = 56\%$	Turnover rate	0.68	0.68	0.68	0.68	0.68	0.68	0.92
	Overall liquidity costs	4.54	4.60	4.73	5.05	6.10	6.46	7.56
	Informational part	0.17	0.19	0.25	0.38	0.68	0.70	0.72
	Non-informational part	4.37	4.40	4.48	4.67	5.42	5.77	6.85
$\alpha_H = 62\%$	Yield spreads	83	93	115	173	397	505	846
$\alpha_L = 53\%$	Turnover rate	0.68	0.68	0.68	0.68	0.78	0.89	1.22
(benchmark)	Overall liquidity costs	4.88	4.99	5.23	5.81	7.53	8.32	10.47
	Informational part	0.51	0.58	0.75	1.14	2.11	2.55	3.62
	Non-informational part	4.37	4.40	4.48	4.67	5.42	5.77	6.85
$\alpha_H = 65\%$	Yield spreads	85	95	118	178	414	529	897
$\alpha_L = 50\%$	Turnover rate	0.68	0.68	0.68	0.68	0.90	1.00	1.68
	Overall liquidity costs	5.26	5.42	5.78	6.66	9.44	10.68	13.92
	Informational part	0.89	1.01	1.30	1.99	4.02	4.91	7.07
	Non-informational part	4.37	4.40	4.48	4.67	5.42	5.77	6.85

Notes: This table shows the effects of the difference between the recovery rates of high-type firms and low-type firms, that is, $\alpha_H - \alpha_L$, on yield spreads (bps), turnover rates (yearly), overall liquidity costs (%), informational liquidity costs (%), and non-informational liquidity costs (%) across different rating classes.

correlate with the illiquidity of their corporate bond portfolio and have pricing implications in the bond market. Suppose we incorporate this clientele effect into our model by assuming that investors differ in their future demands for liquidity. In that case, investors with higher liquidity-shock intensities should trade bonds with lower credit risks (i.e., investment-grade bonds), and investors with higher liquidity-shock intensities should trade bonds with higher credit risks (i.e., high-yield bonds). Such an extension can improve the model's quantitative performance in explaining the high turnover rate for the first yield-spread bin in Table 1.

5.3 Effects of the Recovery Rate Difference

Table 5 presents the comparative-static results concerning $\alpha_H - \alpha_L$, the recovery rate difference between a high-type and a low-type firm. When conducting this analysis, we change the recovery rate difference while holding the average recovery rate $\pi\alpha_H + (1-\pi)\alpha_L$ fixed at 55%. Therefore, an increase in the recovery rate difference raises the degree of information asymmetry between bond investors in the model.

As in Table 5, the effect of the recovery rate difference on the turnover rate in the bond market varies across different rating groups. For representative bonds in the investment-grade region, an increase in the recovery rate difference around its calibrated value has little impact on their turnover rates. This is because informed investors find it unprofitable to exploit their informational advantages on these low-risk bonds. For representative bonds in the speculative-grade region, an increase in the recovery rate difference causes increases in turnover rates by encouraging more informed trading.

Besides, Table 5 shows that an increase in the recovery rate difference tends to raise the size of informational liquidity costs and the yield spread in the bond market. This result is straightforward: an increase in the recovery rate difference increases the degree of information asymmetry in the bond market, resulting in higher informational liquidity discounts in bond pricing. Further, Table 5 shows that the effects of recovery rate difference on informational illiquidity and yield spreads are higher for bonds with higher credit risks.

Note that a change in the recovery rate difference affects bond pricing by changing the information structure in the bond market. So, we can interpret it more broadly as a shock that changes the degree of information asymmetry between bond investors, such as a regulatory reform that changes the accounting transparency of corporate bonds or a policy shock that affects the informativeness of credit ratings. In this regard, our model predicts that an improvement in accounting transparency or credit-rating informativeness can reduce yield spread by lowering informational liquidity costs. However, its effect on trading volume is ambiguous and varies across bonds with different credit risks.

6 Alternative Specification with Direct Information Disclosure

In this section, we consider an alternative setting in which liquidity-shocked bondholders have an option to disclose the information about their firm's type directly. We model this option using linear disclosure costs rather than quadratic disclosure costs for tractability. Specifically, we assume that each bond seller can credibly disclose the true type of her firm with a probability $\theta \in [0, \bar{\theta}]$ at costs of $\delta\theta$, where δ is a constant representing the proportional disclosure costs. If we consider quadratic costs as in the main model, a closed-form solution does not seem to be achievable. For clarification, we do not consider the liquidity-status disclosure channel in this section.

When a bond seller discloses her firm's type, $k \in \{H, L\}$, she can sell her bond at $D^k(x)$, which is the true value of the bond. Otherwise, she can sell the bond at a pooling price of $P(x)$, which will be described in more detail below. The price $P(x)$ conceptually corresponds to the price $P^A(x)$ in our original model because both prices are the price that arises when neither the firm's type nor the seller's liquidity status is known.

In this model, liquidity-shocked bondholders of only high-type firms have incentives to disclose the information about their firms. That is, the information disclosure strategy must depend on a firm's type as well as its fundamental in this setting. In this regard, for each $k \in \{H, L\}$, let $\theta^k(x)$ denote the information disclosure strategy of liquidity-shocked bond sellers of k-type of firms to avoid notational confusion, although $\theta^L(x)$ is constantly zero. The information disclosure strategy of liquidity-shocked bondholders of high-type firms is equal to

$$\theta^H(x) = \begin{cases} \bar{\theta}, & \text{if } D^H(x) - P(x) \geq \delta \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

due to linear disclosure costs.

Non-liquidity-shocked bondholders behave the same way as in the original model. That is, the informed bond selling strategy of non-liquidity-shocked bondholders of low-type firms, $m(x)$, still satisfies the following condition:

$$\begin{cases} m(x) \in [0, \infty), & \text{if } P(x) - \kappa = D^L(x) \\ m(x) = 0, & \text{if } P(x) - \kappa < D^L(x). \end{cases} \tag{14}$$

In addition, the pooling price of bonds, $P(x)$, is now given by

$$P(x) = \frac{\pi\xi(1 - \theta^H(x))D^H(x) + (1 - \pi)(\xi + m(x))D^L(x)}{\pi\xi(1 - \theta^H(x)) + (1 - \pi)(\xi + m(x))},$$

because a fraction $\theta^H(x)$ of liquidity-shocked bond sellers of high-type firms reveal their

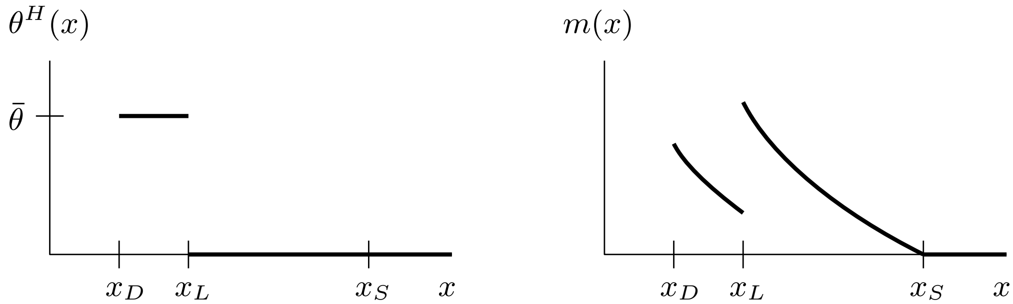

Figure 7: The left panel plots the typical shapes of $\theta^H(x)$ and the right panel plots the typical shape of $m(x)$ in the model with direct information disclosure. We omit presenting the trivial case of $x_D = x_L$, which means that no liquidity-shocked bond sellers reveal their private information about their firms in this setting.

private information, while none of the liquidity-shocked bond sellers of low-type firms reveal their private information.

Moreover, the HJB equations for $D^H(x)$ and $D^L(x)$ are now described as

$$rD^H = c + \xi \left[\max_{\theta^H(x) \in [0,\bar{\theta}]} \theta^H(x)(D^H(x) - P(x) - \delta) + P(x) - \kappa - D^H(x) \right] + \mathcal{A}D^H \tag{15}$$

and

$$rD^L = c + \xi(P(x) - \kappa - D^L(x)) + \max_{m(x) \geq 0} m(x)(P(x) - \kappa - D^L(x)) + \mathcal{A}D^L, \tag{16}$$

subject to the same boundary conditions as those in the original model. Here, we have omitted writing the information disclosure option for liquidity-shocked bondholders of low-type firms because these bondholders never reveal their private information about the firm's type.

In this model, we still have two thresholds x_S and x_L such that (i) non-liquidity-shocked bondholders of low-type firms start to sell their bonds for the informational reason when their firm's fundamental x_t falls below x_S and (ii) liquidity-shocked bondholders of high-type firms reveal their private information with the maximum capacity $\bar{\theta}$ when their firm's fundamental x_t is below x_L, as shown in the left panel of Figure 7. Here, note that the information disclosure strategy $\theta^H(x)$ jumps up from 0 to $\bar{\theta}$ when the firm's fundamental hits x_L from above, because bond sellers face linear costs when disclosuing their private information. We pin down the equilibrium pair of thresholds, x_L and x_S, of this model in Appendix A.3. Additionally, to ensure that x_L lies below x_S in equilibrium, we impose the following condition:

$$\kappa < \pi(\kappa + \delta). \tag{17}$$

If this condition does not hold, liquidity-shocked bond sellers of high-type firms would be willing to disclose their information about the firms even before the market suffers from informed-bond selling, which does not seem to be an interesting case.

Notably, in this model, since the information disclosure strategy $\theta^H(x)$ jumps up when the firm's fundamental hits x_L from above, the informed bond selling strategy $m(x)$ jumps down to some extent at that point, because an increase in $\theta^H(x)$ means that the number of liquidity-shocked bond sellers of high-type firms included in the entire pool of liquidity-shocked bond sellers has dropped. When the firm's fundamental falls further, $m(x)$ rebounds because the number of liquidity-shocked bond sellers of high-type firms, who disclose their private information, does not increase any more. The right panel of Figure 7 illustrates this property. In this figure, we omit the corner case of $x_L = x_D$ because such a case in this alternative model is identical to the case of $\delta = \infty$ in the original model.

Note that the qualitative property of $m(x)$ is similar to that exhibited in the original model. However, since $m(x)$ shows a discontinuous jump, the bond turnover rate, which is again equal to $\xi + m(x)$, will also exhibit a downward jump. As this outcome is not perfectly aligned with the pattern observed in data, we do not adopt the specification with direct information disclosure. If we assume quadratic disclosure costs, we may obtain continuous turnover rates. However, as mentioned before, this model does not permit closed-form solutions.

7 Conclusion

In this paper, we develop a structural credit-risk model to study the effect of information asymmetry on yield spreads and trading volume in the corporate bond market. Due to information asymmetry between bond buyers and sellers, the model predicts a non-monotonic relationship between bonds' turnover rates and yield spreads, consistent with US corporate bond trading data. In the calibrated model, we find that the effects of information asymmetry on bond prices are non-negligible for investment-grade bonds and are economically sizable for speculative-grade bonds.

Our paper highlights the critical role of informational frictions in shaping corporate bond trading volume and prices. The model made several simplifying assumptions about the trading mechanism. First, we model the presence of non-informational frictions by introducing a reduced-form transaction cost in an otherwise competitive market. In reality, these non-informational frictions appear as search costs, inventory costs, and dealers' market power. Thus, future works that endogenize these non-informational frictions in our model can provide further insights into the interaction between informational and non-informational frictions in corporate bond markets.

Besides, we allow liquidity-shocked bondholders to reveal their liquidity status with costly efforts. The liquidity-status revealing option captures the process of bond investors searching for trustworthy dealers or trading venues to reduce informational illiquidity. Micro-founding the liquidity-status revealing option, especially by incorporating time delay as a potential source of signaling device and indirect trading costs (Daley and Green, 2012), can deepen our understanding of information-theoretic illiquidity in corporate bond markets.

A Appendix

A.1 Proof of Proposition 3.1

In this section, we complete the proof of the existence and uniqueness of equilibrium. We first note that

$$m(x)(P^A(x) - \kappa - D^L(x)) = 0, \quad \forall x \geq x_D, \tag{18}$$

because the second condition in (10) implies $P^A(x) - \kappa - D^L(x)$ has to be 0 unless $m(x) = 0$. Due to this property, by subtracting equation (4) from equation (3), we obtain

$$(r + \xi)J(x) = \mathcal{A}J(x) \tag{19}$$

subject to $J(x_D) = \frac{(\alpha_H - \alpha_L)x_D}{r-\mu}$ and $\lim_{x\to\infty} J(x) = 0$. The explicit solution for $J(x)$ is given by

$$J(x) = \frac{(\alpha_H - \alpha_L)x_D}{r-\mu}\left(\frac{x}{x_D}\right)^{\phi}, \tag{20}$$

where $\phi = \frac{-\mu+\frac{\sigma^2}{2}-\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2\sigma^2(r+\xi)}}{\sigma^2} < 0$. Here, there are no non-homogeneous terms. Hence, as $J(x_D) > 0$ and $\lim_{x\to\infty} J(x) = 0$, the probabilistic representation for $J(x)$ implies $J(x)$ must be decreasing in x. Then there must be a unique pair of (x_L, x_S) that satisfies the conditions in (11) and (12), respectively. Furthermore, we can easily see that $x_L < x_S$ because $J(x)$ is decreasing in x and $\delta\bar{\theta} > 0$. The pair of equilibrium thresholds (x_L, x_S) can be computed numerically, using the explicit expression of $J(x)$.

To complete equilibrium characterization, we now prove the global optimality conditions that must be satisfied by the liquidity-status revealing strategy $\theta(x)$ and the informed bond-selling strategy $m(x)$. To this aim, we claim the following conditions hold:

$$\begin{cases} P^R(x) - P^A(x) \geq \delta\bar{\theta}, & \forall x \in [x_D, x_L) \\ P^R(x) - P^A(x) \in [0, \delta\bar{\theta}], & \forall x \in [x_L, x_S) \\ P^R(x) - P^A(x) = 0, & \forall x \in [x_S, \infty) \end{cases} \tag{21}$$

and

$$\begin{cases} P^A(x) - \kappa = D^L(x), & \forall x \in [x_D, x_S) \\ P^A(x) - \kappa \leq D^L(x), & \forall x \in [x_S, \infty). \end{cases} \tag{22}$$

The first condition in (21) holds because $P^R(x) - P^A(x) = \pi J(x) > \delta\bar{\theta} + \kappa > \delta\bar{\theta}$ for all $x < x_L$. This condition justifies the conjectured solution that $\theta(x) = 1$ for all such x. The second condition in (21) holds because $0 \leq P^R(x) - P^A(x) = \pi J(x) - \kappa \leq \delta\bar{\theta}$ for all $x \in [x_L, x_S)$. This condition justifies the conjectured solution that $0 \leq \theta(x) < \bar{\theta}$ for all such x. We have already shown the third condition in (21) when deriving the second line of (10). This condition justifies the conjectured solution that $\theta(x) = 0$ for all $x \geq x_S$.

Regarding the informed bond-selling strategy $m(x)$, we have already shown the first condition in (22) when deriving the first line of (10). This condition justifies the conjectured solution that $m(x) \geq 0$ for all $x \in [x_L, x_S)$. The second condition in (22) holds because

$P^A(x) - D^L(x) = \pi J(x) \leq \kappa$ for all $x \geq x_S$. This condition justifies the conjectured solution that $m(x) = 0$ for all such x. We have therefore completed characterizing the equilibrium of our model.

A.2 Closed-form Solutions

In this section, we provide the closed-form solution for our model for any given pair of (x_L, x_S) such that $x_L < x_S$. To begin, having pinned down $J(x)$, x_L, and x_S, we can now solve the HJB equation for $D^L(x)$. Specifically, due to the properties (10) and (18), the HJB equation (4) can be rewritten as

$$rD^L = c + \xi\left[1_{x \leq x_L}\left(\bar{\theta}\pi J - \bar{\theta}\kappa - \frac{\delta\bar{\theta}^2}{2}\right) + 1_{x_L < x < x_S}\frac{(\pi J - \kappa)^2}{2\delta} + 1_{x_S \leq x}(\pi J - \kappa)\right] + \mathcal{A}D^L. \quad (23)$$

The boundary conditions for the low-type firm's debt value are given by

$$D^L(x_D) = \frac{\alpha_L x_D}{r - \mu}, \lim_{x \uparrow x_L} D^L(x) = \lim_{x \downarrow x_L} D^L(x), \lim_{x \uparrow x_L} D^L_x(x) = \lim_{x \downarrow x_L} D^L_x(x), \quad (24)$$

$$\lim_{x \uparrow x_S} D^L(x) = \lim_{x \downarrow x_S} D^L(x), \lim_{x \uparrow x_S} D^L_x(x) = \lim_{x \downarrow x_S} D^L_x(x), \quad (25)$$

which are the standard value-matching and smooth-pasting conditions.

To provide the closed-form solution of $D^L(x)$, let $J_1 = J(x)/x^\phi$ for notational convenience. Then the closed-form solution of $D^L(x)$ is given by

$$D^L(x) = \begin{cases} \frac{c - \xi\bar{\theta}\kappa - \frac{\xi\delta\bar{\theta}^2}{2}}{r} + \frac{\xi\bar{\theta}\pi J_1}{l(\phi)}x^\phi + A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x_D \leq x < x_L \\ \frac{c + \frac{\xi\kappa^2}{2\delta}}{r} + \frac{\xi}{2\delta}\left(\frac{\pi^2 J_1^2}{l(2\phi)}x^{2\phi} - \frac{2\pi J_1\kappa}{l(\phi)}x^\phi\right) + A_3 x^{\eta_1} + A_4 x^{\eta_2}, & \text{if } x_L \leq x < x_S \\ \frac{c - \xi\kappa}{r} + \frac{\xi\pi J_1}{l(\phi)}x^\phi + A_5 x^{\eta_2}, & \text{if } x_S \leq x, \end{cases} \quad (26)$$

where $l(a) := r - a\mu - a(a-1)\sigma^2/2$ for any a. The coefficients A_1, ..., A_5 are determined from the boundary conditions described in (24).

Using the above results, we can readily pin down the remaining equilibrium objects. That is, $D^H(x)$ is given by $D^L(x) + J(x)$; $P^R(x)$ is given by $\pi D^H(x) + (1-\pi)D^L(x)$; $P^A(x)$

is given by the expression in (10); and $\theta(x)$ and $m(x)$ are given by

$$\begin{cases} \theta(x) = \bar{\theta}, & \text{if } x \in (x_D, x_L] \\ \theta(x) = \frac{\pi J(x)-\kappa}{\delta}, & \text{if } x \in (x_L, x_S] \\ \theta(x) = 0, & \text{if } x \in (x_S, \infty) \end{cases}$$

and

$$\begin{cases} m(x) = \frac{\pi\xi(1-\theta(x))(D^H(x)-P^A(x))+(1-\pi)\xi(1-\theta(x))(D^L(x)-P^A(x))}{(1-\pi)(P^A(x)-D^L(x))}, & \text{if } x \in (x_D, x_S] \\ m(x) = 0, & \text{if } x \in (x_S, \infty), \end{cases}$$

respectively, due to (5), (8), and (10). This expression confirms that $0 < \theta(x) < \bar{\theta}$ and $m(x) > 0$ for all $x \in (x_L, x_S)$ because $\kappa < \pi J(x) < \kappa + \delta\bar{\theta}$ for all such x as shown above.

A.3 Equilibrium Characterization for the Alternative Model

In this section, we characterize the equilibrium of the extended model in Section 6. To this aim, let $J(x)$ be $D^H(x) - D^L(x)$ as before. From the properties in (13) and (6), we see that our goal must be to find a pair of thresholds, x_L and x_S, such that

$$x_L < x_S, \quad J(x_L) = \kappa + \delta, \quad \pi J(x_S) = \kappa.$$

Here, we have ignored the corner case of $x_L = x_D$ because analyzing this case is trivial as in the model with the liquidity-status revealing option.

Then, note that from the HJB equations for $D^H(x)$ and $D^L(x)$ in (15), $J(x)$ must satisfy

$$rJ(x) = \xi 1_{x<x_L}\bar{\theta}(J(x) - \kappa - \delta) - \xi J(x) + \mathcal{A}J, \tag{27}$$

subject to $J(x_D) = \frac{(\alpha_H-\alpha_L)x_D}{r-\mu}$, where we have used the property in (13) and the fact that $P(x) - \kappa = D^L(x)$ for all $x \le x_S$. To emphasize the fact that $J(x)$ depends on a given threshold x_L, we denote this value function as $J(x; x_L)$.

We now claim that there is a unique threshold x_L such that $x_D < x_L$ and $J(x_L; x_L) = \kappa + \delta$. Note that this problem is equivalent to showing that the solution to the following HJB equation has a unique point x_L such that $J(x_L) = \kappa + \delta$:

$$rJ(x) = \xi \max\{\bar{\theta}(J(x) - \kappa - \delta), 0\} - \xi J(x) + \mathcal{A}J,$$

subject to $J(x_D) = \frac{(\alpha_H - \alpha_L)x_D}{r-\mu}$. Here, we have used the same notation $J(x)$ to avoid abuse of notation.

Now, note that since $\lim_{x\to\infty} J(x) = 0$, there should be at least one such point. By way of contradiction, suppose there are more than one such points. Then, again, since $\lim_{x\to\infty} J(x) = 0$, there must be a point z such that

$$J(z) > \kappa + \delta, \quad J_x(z) = 0, \quad J_{xx}(z) < 0.$$

Then the HJB equation in (27) implies

$$(r + \xi)J(z) < \xi\bar{\theta}(J(z) - \kappa - \delta),$$

which leads to $J(z) < 0$ because $\bar{\theta} < 1$, a contradiction. Hence, there is a unique point x_L such that $J(x_L) = 0$.

Using a similar argument, we can further show that $J(x)$ is strictly decreasing in x. Then, due to the parameter condition in (17), there is a unique threshold x_S such that $x_L < x_S$ and $\pi J(x_S) = \kappa$, completing the proof of our claim.

Following the arguments used Appendix A.1 similarly, we can also show the global optimality of the information disclosure strategy and informed bond-selling strategy. Moreover, we can pin down all equilibrium objects in closed form for any given pair of (x_L, x_S), such that $x_L < x_S$. But we omit this task for brevity.

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