Optimal Credit History Disclosure: What Should Be Disclosed?*

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Abstract

We develop a dynamic model of unsecured debt contracts under adverse selection, where an entrepreneur borrows to fund a project with returns dependent on both entrepreneurial and aggregate productivity. Entrepreneurial productivity is private information. Lenders assess the entrepreneur's productivity based on their credit history, which may include default and/or transaction histories, along with historical aggregate productivity. While transaction history is more informative than default history, its disclosure is effective only when current economic conditions are favorable and it incurs higher social costs. We explore how future economic outlooks influence the effects of credit history disclosure and identify the optimal information regime.

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1 Introduction

Credit reports are vital for lenders evaluating borrowers' credit risk, while historical economic conditions also play a critical role in assessing repayment capacity. Despite their importance in real credit markets, research on the combined use of credit reports and economic conditions for credit risk assessment remains scarce. Furthermore, credit reports do not disclose transaction history, referring to the specific terms of past debt contracts, though they detail default history.[1] Consequently, the academic exploration of the economic consequences of disclosing transaction history is limited, even though Decentralized Finance (DeFi) makes such disclosure feasible.[2] Thus, the following questions remain unanswered: How do lenders incorporate a borrower's credit history into risk assessment? How does the visibility of credit history affect current debt contract terms? How does the type of disclosed credit information influence debt contracts and economic activity? What constitutes the optimal credit history disclosure scheme in credit markets?

We address these questions by developing a dynamic model of unsecured debt contracts with adverse selection. We examine how borrowers' operational history, credit history—including default and/or transaction history—and past aggregate economic conditions are used to assess credit risk. We investigate how the accessibility of different types of borrower credit history information influences equilibrium outcomes. Additionally, we explore the dynamics of borrowing costs across various information regimes and examine the optimal level of credit history disclosure.

The model features an entrepreneur who borrows an investment good from lenders to undertake a project, with returns depending on both entrepreneurial and aggregate productivity. Aggregate productivity is a random variable that is independent across periods. There are two types of entrepreneurs: high productivity (H-type) and

[1]In the U.S., credit reports are issued by credit bureaus like Equifax and Experian. They detail payment timeliness, instances of late payments, and any history of being sent to collections or charged off. However, these reports do not disclose the particular contracts chosen by an individual borrower (see Elul and Gottardi (2015)).

[2]DeFi refers to a set of financial services built on blockchain technology, in which all transactions are recorded on a blockchain (see Chiu et al. (2022), Gramlich et al. (2022) and Jiang et al. (2023) for a detailed discussion on DeFi and its economic implications). DeFi platforms can be designed to allow lenders to access specific historical information about borrowers, such as default history and transaction records. For example, platforms like Aave operate on public blockchains where all transactions are publicly recorded and accessible, ensuring transparency of individual transaction details. However, platforms such as Maple Finance do not provide comprehensive credit histories, such as the specific terms of all debt contracts previously entered by a borrower.

low productivity (L-type), with their specific productivity being private information known only to themselves.

In the model, past realized aggregate productivity is public information, akin to the availability of GDP data in most countries. Similarly, an entrepreneur's operational history—whether they previously ran the project—is also publicly accessible. Lenders can observe the entrepreneur's credit history, which details debt contract terms and/or default decisions, depending on the information regime. Consequently, lenders can leverage historical aggregate productivity and the entrepreneur's past economic decisions to assess the entrepreneur's productivity and, hence, credit risk.

We examine four distinct information regimes: 1) the no-information regime, where lenders cannot observe any aspect of the entrepreneur's credit history, 2) the default history regime, where lenders can only access the entrepreneur's default history, 3) the transaction history regime, where lenders can view the terms of the entrepreneur's past contracts, and 4) the full-information regime, where lenders have access to both the transaction and default histories.

In the model, the H-type can secure more favorable future contract terms by establishing a distinct credit history from the L-type, as lenders can correctly infer the true type through an investigation of credit history. Specifically, transaction history reveals the true type whenever different contracts are entered into by each type. Thus, the H-type has more incentive to offer a separating contract with terms distinct from those of the L-type when transaction history is observable. Crucially, the H-type offers such a separating contract only if transaction history is observable; disclosing default history alone is insufficient to support a separating equilibrium. Consequently, transaction history can work as a signaling device, whereas default history cannot. This suggests that transaction history disclosure increases loan rate dispersion, even among first-time borrowers, providing a theoretical prediction testable in the DeFi industry.

However, such a separating equilibrium, where each type engages in distinct contracts, is feasible only if the aggregate productivity in the current period is sufficiently high on average. Otherwise, the L-type, once identified, will be unable to secure the investment good and will thus mimic the H-type unless doing so yields no benefits. Thus, disclosing transaction history impacts real allocations only when the expected aggregate productivity is sufficiently high in the current period.

Conversely, default history can be informative even when the expected aggregate productivity is not sufficiently high. Since the H-type has a lower default risk than the L-type, each type may make different default decisions given aggregate productivity. In such cases, future lenders can infer the entrepreneur's true type by observing their default history and past aggregate productivity. Consequently, in the default history regime, as in traditional credit markets, default experiences increase future borrowing costs, while those without defaults generally face declining borrowing costs over time, consistent with earlier empirical findings (e.g., Berger and Udell (1995), Bharath et al. (2011), and Peri and Rachedi (2019)). Furthermore, the cost for the H-type to differentiate from the L-type through a separating contract is lower under a full-information regime than under the transaction history regime, as the observability of default history discourages the L-type from mimicking.

In terms of welfare, it is socially optimal for the L-type to run their project if its expected return is high enough to secure funding despite type disclosure.[3] In this case, a pooling equilibrium, where both types engage in the same debt contract, minimizes aggregate liquidation costs from defaults, which are social costs. Consequently, welfare is higher when the type is not disclosed. Conversely, if the expected return on the L-type's project is low, disclosing the entrepreneur's true type enables lenders to withhold investment from the unproductive L-type, thereby enhancing welfare. Additionally, such disclosure can prevent market collapse caused by severe adverse selection.

Consequently, when the economy expects high aggregate productivity in the future, concealing the entrepreneur's type enhances social welfare, making the no-information regime—characterized by withholding both default and transaction histories—optimal. Conversely, if economic downturns are anticipated, disclosing credit histories becomes preferable. The specific optimal level of credit history disclosure depends on the current economic conditions.

First, when current economic prospects are poor, the H-type cannot offer a separating contract. In this case, disclosing transaction history is irrelevant, whereas default history, combined with aggregate productivity history, can reveal the entrepreneur's true type whenever default decisions differ by type given the aggregate productivity. Conversely, when current economic prospects are favorable, disclosing

[3] In the model, when the credit market is controlled by a monopolistic DeFi platform, social welfare aligns directly with the platform's profit. Consequently, the information regime that maximizes social welfare is the one that also maximizes the platform's profit.

transaction history enables the H-type to offer a separating contract, allowing future lenders to always verify the entrepreneur's true type. Consequently, the full information regime can be optimal.[4]

However, disclosing transaction history incurs higher social liquidation costs than disclosing default history in the current period. Therefore, the full information regime is optimal only if the economy faces a sufficiently severe adverse selection problem in the future. As future average aggregate productivity increases and/or the proportion of H-types rises, the optimal information regime shifts from the full information regime to the default history regime, and eventually to the no information regime. Conversely, as current average aggregate productivity increases, default history becomes less informative due to the rarity of defaults, making the full information regime more likely to dominate the default history regime.

Literature review This paper adds to the tradition of studying adverse selection problems in credit markets. While most existing literature focuses on one-time transactions in a single-period model, we construct a dynamic model that allows us to investigate reputation formation in credit markets.[5] Hennessy et al. (2010), Morellec and Schürhoff (2011), and Strebulaev et al. (2016) develop dynamic models of credit market with adverse selection, but these papers essentially reduce the signaling problem to a static one by assuming private information is short-lived or that debt is a one-time decision in a real options context. In contrast, our model accommodates a broader range of signaling strategies and explores the dynamics of reputation formation and the evolution of lenders' beliefs over time.

Ordoñez et al. (2019) go a step further by studying secured loan markets with asymmetric information and shows that good borrowers can signal their credit quality through repayment history when collateral value uncertainty is low. However, the significance of a borrower's credit history in collateralized credit markets differs from that in unsecured markets, as collateral mitigates lender's losses from defaults and

[4]The H-type can offer a separating contract under the transaction history regime as well, but the L-type has less incentive to mimic when both default and transaction histories are observable compared to when only transaction history is observable. Consequently, the H-type can offer a separating contract at a lower cost under the full information regime, which results in lower social costs associated with defaults compared to the transaction history regime.

[5]See Bester (1985), Besanko and Thakor (1987), Figueroa and Leukhina (2015), Jaffee and Russell (1976), and Milde and Riley (1988) for single period models of credit market with adverse selection for instance.

can itself serve as a signaling device (see Bester (1985)). Chatterjee et al. (2023) and Diamond (1989) examine reputation formation through default history, while Bhaskar and Thomas (2019), Blattner et al. (2023), and Elul and Gottardi (2015) show that coarsening default history can enhance welfare.[6]

Our paper advances the study of credit history in unsecured credit markets in three ways. First, we incorporate aggregate shocks to examine how lenders leverage historical economic conditions alongside individual borrower histories to assess credit risk, providing insights into the enduring influence of macroeconomic conditions on interest rates and credit access.[7] Second, we investigate how future economic outlooks affect the impact of disclosing credit history. Third, we extend the analysis to include transaction history, a key element in DeFi platform design. Specifically, we examine its role as a signaling device—contrasting with prior studies focused on pooling equilibria—and explore optimal credit history disclosure in markets.[8]

The rest of this paper is organized as follows. Section 2 outlines the economic environment of the model. Section 3 details the game structure between a borrower and a lender, and section 4 characterizes the equilibrium. In section 5, we conduct a welfare analysis to determine the optimal information regime. Section 6 concludes.

2 Model

In this section, we set up the model.

Physical environment The economy consists of two dates, $t = 1, 2$, and each period t is divided into two subperiods: morning and afternoon. Morning is the investment period and consumption occurs in the afternoon. The actors in the model are an entrepreneur and two lenders 1 and 2. The entrepreneur lives for both periods with the discount factor $\beta \in (0, 1)$ across periods, lender 1 lives in period 1, and

[6] Boot and Thakor (1994) examines the dynamics of loan interest rates in credit markets with a moral hazard problem, revealing a decline in loan interest rates over time. However, our focus remains on addressing adverse selection problems in credit markets. Dari-Mattiacci and Gottardi (2024) investigate the optimal policy for expunging criminal history in the labor market.

[7] Sharma et al. (2024) investigate the influence of macroeconomic determinants on credit risk in the UK banking sector, revealing how past economic conditions can exert a lasting impact on borrowing interest rates and credit availability.

[8] Kawai et al. (2022) highlight the significance of signaling devices in improving market efficiency and welfare in online credit markets, particularly under prevalent adverse selection.

lender 2 lives in period 2. Thus, the entrepreneur faces a different lender in each period. The instantaneous utility of all agents in each period equals the quantity of consumption in the afternoon, i.e., agents have a constant marginal utility of 1.

Lender t receives an indivisible endowment of one unit of an investment good in the morning in period t. The investment good can be either lent to the entrepreneur or invested in a saving technology that yields a certain return of $\gamma > 0$ units of the consumption good in the afternoon. The entrepreneur does not receive any endowments in the morning. Instead, the entrepreneur has access to an investment project that produces w_t units of consumption goods in the afternoon in each period $t \in \{1, 2\}$ if the project is funded with one unit of the investment good in the morning, and produces zero units otherwise. The outcome, w_t, of the project depends on the aggregate productivity, A_t, and the entrepreneurial productivity θ, as $w_t = A_t\theta$. The aggregate productivity A_t is uniformly distributed with the support of $\left[0, \overline{A}_t\right]$ in period $t \in \{1, 2\}$ and it is independent across periods. The entrepreneur has productivity θ_H with the probability $\sigma \in (0, 1)$ and has θ_L, where $0 < \theta_L < \theta_H$, with the complement probability. The entrepreneurial productivity is realized at the beginning of the morning in period 1 and remains fixed until the end of period 2. We refer to the entrepreneur with θ_H as the H-type and the one with θ_L as the L-type, respectively.

We assume that the distributions for A_t and probability σ are public information. However, entrepreneurial productivity θ is the entrepreneur's private information. Furthermore, the aggregate productivity A_t is not observable in the afternoon in period t. Thus, only the entrepreneur can observe the exact realized return of his/her project.

Borrowing with a debt contract In the model, the entrepreneur must borrow the investment good from lender t to run his/her project in the morning in period $t = 1, 2$. We assume that the entrepreneur offers a contract to lender t in each period and lender t either accepts or rejects the offer.[9]

The contract between the entrepreneur and lender t is assumed to be a debt

[9] Alternatively, the contracting problem between borrower and lender can be viewed as a bargaining problem in over-the-counter markets. This framework assumes a continuum of borrowers and lenders, each with unit mass, randomly matched each morning in every period—a standard approach in over-the-counter market models (See Duffie et al. (2005), Berentsen et al. (2014), Mattesini and Nosal (2016), and Herrenbrueck and Geromichalos (2017)).

contract. Although we focus on a debt contract, a debt contract often emerges as the optimal contract when project returns are unobservable with private information because equity contracts are infeasible. For example, Gale and Hellwig (1985) and Townsend (1979) show that unobservability implies that contracts are optimal for the debt form. In our setup, it can be shown that a debt contract is optimal if the exact amount of repayment that the entrepreneur made at $t = 1$ is unobservable in period $t = 2$.[10]

A debt contract is described by the repayment $r_t \in \mathbb{R}_+$ at time t given the fixed loan size. We assume that a debt contract involves a commitment to liquidation for all payments less than r_t and liquidation implies the destruction of all output from the project the same as in Diamond (1989). Thus, if the project's return is lower than r_t, the entrepreneur has no option but to default. Note that the entrepreneur of type $i \in \{H, L\}$ would inevitably default on a contract $r_t > \overline{A}_t\theta_i$, resulting in zero gains. Thus, the entrepreneur will not offer such a contract $r_t \geq \overline{A}_t\theta_i$ unless entering into the contract r_t in period t provides any additional non-pecuniary benefits.[11] In the following analysis, we say that contract r' is lower than r'' if $r' < r''$. Furthermore, we assume that if the entrepreneur decides not to run a project in period t, he/she offers a contract $r_t = 0$, which will be rejected by lender t with certainty, without loss of generality.

A model of unsecured credit often assumes that borrowers face limited access to the credit market following defaults, and there have been extensive studies on how the severity of penalties for defaulters affects equilibrium allocations (e.g., Azariadis and Kass (2013) and Kehoe and Levine (1993)). In contrast, our focus lies in the channel through which a borrower's history of economic decisions in the past influences the terms of debt contracts and real allocations. To focus on the main issue, we assume

[10]To elaborate this argument, note that the repayment on any equity contract in the model economy must depend on the information provided by the entrepreneur because only the entrepreneur can observe the exact realized return from the project. Specifically, a contract defines a repayment function $R_1(w_1')$ such that the entrepreneur pays $R_1(w_1')$ units of consumption goods in period 1 after reporting a signal w_1' about the output from the project to lender 1. Now suppose that lender 2 cannot observe the exact signal w_1' and repayment that the entrepreneur made in period 1 although lender 2 can observe the terms of contract $R_1(\cdot)$. Then, the entrepreneur will always choose w_1' so as to minimize the payment to the lender in period 1 whenever he/she decides to honor the contract. Thus, the payment is constant and, hence, the contract has the form of the debt contract similar to results in Jang and Kang (Forthcoming) and Williamson (1986).

[11]Here, we assume that if the entrepreneur is indifferent between running a project with a contract $r_t \geq \overline{A}_t\theta_i$ and not running the project, the entrepreneur chooses not to run the project.

that there is no penalty for defaulters. Thus, the entrepreneur can access the credit market in period 2 following defaults in period 1. However, the main implications of the model remain unchanged even when we assume that the entrepreneur can meet lender 2 to borrow the investment good in period 2 with a probability $\rho \in (0,1)$, following a default in period 1.[12]

History information In the model economy, there are three types of histories that lender 2 can use to evaluate the entrepreneur's credit risk.

First, many countries in the world release time-series data on gross domestic production (GDP) and total factor productivity (TFP) to the public. We incorporate this reality into the model in the following way: In the morning in period 2, all agents, including lender 2, can observe the realized aggregate productivity A_1 (common prouctivity history). Note that the realized A_t is not observable in the afternoon in period $t \in \{1,2\}$, which is also consistent with the real-world observation that GDP and TFP data being published with a lag.

Second, we assume that lender 2 can observe the entrepreneur's operation history. To clarify, let $o = 1$ if the entrepreneur runs the project in period 1 and $o = 0$ otherwise. Then, $o \in \{0,1\}$ summarizes the entrepreneur's operation history and we assume that variable $o \in \{0,1\}$ is the public information in the morning in period 2. Therefore, lender 2 can observe whether the entrepreneur ran his/her project in period 1.

Finally, the last type of history information is the entrepreneur's credit history. To make the definition of credit history concrete, suppose that the entrepreneur entered into a debt contract r_1 with lender 1 in period 1, and let $d = 1$ if the entrepreneur defaults on contract r_1 and $d = 0$ otherwise. Then, r_1 and d capture the entrepreneur's transaction history and default history, respectively, that the entrepreneur made in period 1.[13] Collectively, the pair $\{r_1, d\}$ constitutes the entrepreneur's intrinsic credit history. The observed credit history, denoted by ω, is a subset of this intrinsic credit history, $\{r_1, d\}$, determined by the information disclosure regime.

In traditional credit markets, lenders can access a borrower's default history

[12]Specifically, if there is a penalty on defaulters, there are additional requirements for pooling and separating equilibrium without a break to exist in section 4. Otherwise, the main implications do not change.

[13]Note that if the entrepreneur does not run the project in period 1, then the entrepreneur has no intrinsic credit history, and, in this case, we let $r_1 = d = 0$.

through credit reports but not their transaction history. However, recent advancements in blockchain technology and smart contracts allow DeFi platforms to disclose specific borrower information, such as default history and transaction records. Motivated by these distinctions, we explore all possible scenarios for information disclosure regimes:

1. (No-information regime) Lender 2 can observe no information about the entrepreneur's credit history: $\omega = \varnothing$.

2. (Full-information regime) Lender 2 can observe both the transaction history and default history: $\omega = \{r_1, d\}$.

3. (Default history regime) Lender 2 can observe the default history: $\omega = \{d\}$.

4. (Transaction history regime) Lender 2 can observe the transaction history: $\omega = \{r_1\}$.

Note that the entrepreneur with different intrinsic credit history $\{r_1, d\}$ could have the same credit history ω depending on the information regime and what matters is the credit history ω.

3 Game structure

In this section, we describe the game between the entrepreneur and lender t in each period $t \in \{1, 2\}$, agents' strategy, and lender t's belief system.

Game structure in each period In each period, there is a game between the long-lived entrepreneur and short-lived lenders. A sequence of moves in each period is as follows. In the morning in period t, the entrepreneur offers a contract r_t to lender t. Then, lender t decides whether to accept the offered contract or not. If lender t rejects the offer, the game ends. On the other hand, if lender t accepts the offer, lender t provides the investment good to the entrepreneur and the entrepreneur runs the project in the morning. Then, after observing the return from the project in the afternoon, the entrepreneur decides whether to repay r_t units of consumption goods to lender t or to default.

Agents' strategies To analyze the entrepreneur's strategy, we define I_t for each $t \in \{1, 2\}$ as the set of histories available to lender t in period t: $I_1 = \varnothing$ and $I_2 = \{A_1, o, \omega\}$. Let $\mathbb{I}_t$ denote the set of all feasible I_t. Then, a period $t \in \{1, 2\}$ strategy of the entrepreneur specifies a contract $r_t \in \mathbb{R}_+$ as a function of (θ, I_t) and a set $D_t \subset [0, 1]$ of A_t as a correspondence of (θ, I_t, r_t) such that the entrepreneur defaults on the debt contract r_t if and only if $A_t \in D_t$. Next, the strategy of lender t is an acceptance rule that specifies a set $\mathcal{B}_t \subset \mathbb{R}_+$ of acceptable contracts r_t as a correspondence of I_t.

If there is no risk of confusion, we drop arguments for each decision rule: We use r_t, D_t, and $\mathcal{B}_t$ instead of $r_t(\theta, I_t)$, $D_t(\theta, I_t, r_t)$, and $\mathcal{B}_t(I_t)$, respectively. Further, we use $r_{i,t}$ to denote the equilibrium contract that the type $i \in \{H, L\}$ entrepreneur offers in period $t \in \{1, 2\}$ in what follows.

Belief system Because the entrepreneur's productivity θ is the entrepreneur's private information, lender t must form beliefs about θ before making an acceptance decision for the proposed contract r_t. Specifically, lender t constructs the belief using all available information which includes the terms of the offered contract r_t and the public history set I_t. We write $\mu_t : \mathbb{R}_+ \times \mathbb{I}_t \to [0, 1]$ for the lender's belief function in period $t \in \{1, 2\}$, assigning the probability that the entrepreneur is the H-type.

We impose the following restrictions on the belief μ_t, which we believe are reasonable assumptions. First, we assume that if lender 2 observes that the entrepreneur defaulted on contract r_1 when the realized A_1 was sufficiently high as $A_1 \geq \frac{r_1}{\theta_H}$ so that the H-type entrepreneur could make repayment r_1, then lender 2 believes that the entrepreneur is the L-type. Second, suppose that in period 1, the H-type chooses r', while the L-type chooses $r'' \neq r'$, and that lender 2 can observe the entrepreneur's transaction history in the morning of period 2, i.e., $r_1 \in \omega$. In this case, we assume that lender 2 believes that the entrepreneur is the H-type (and L-type) if lender 2 observes that the entrepreneur offered r' (and r'') in period 1: $\mu_2(\cdot, I_2) = 1$ if $r' \in \omega$ and $\mu_2(\cdot, I_2) = 0$ if $r'' \in \omega$. Similarly, we assume that if the H-type has operation history o while the L-type has $o' \neq o$, then $\mu_2(\cdot, \{A_1, o, \omega\}) = 1$ and $\mu_2(\cdot, \{A_1, o', \omega\}) = 0$.

Optimal strategies In the model, defaults destroy all output from the project and do not increase the probability that lender 2 believes the entrepreneur is the H-type. Consequently, the entrepreneur will always make repayment whenever it is feasible and the optimal default strategy for the entrepreneur is $D_t \in \left[0, \frac{r_t}{\theta}\right)$ for any $r_t > 0$. Then, the entrepreneur's expected payoff from entering into contract r with lender t in period $t \in \{1, 2\}$ is given as

$$u_t(r \mid \theta) = \frac{1}{\overline{A}_t} \int_{\min\left\{r/\theta, \overline{A}_t\right\}}^{\overline{A}_t} (A\theta - r) dA. \tag{1}$$

Note that the economy ends in period 2. Thus, what matters to the entrepreneur is whether he/she can achieve a positive expected return by entering into a contract with lender 2 without consideration of reputation building. Specifically, given lender 2's acceptance rule $\mathcal{B}_2$ and the entrepreneur's public history set $I_2 = \{A_1, o, \omega\}$ in the morning in period 2, the entrepreneur solves

$$V_2(\theta, I_2) = \max_{r \in \mathbb{R}_+} \left\{ \mathbf{1}_{\mathcal{B}_2(I_2)}(r) u_2(r \mid \theta) \right\}, \tag{2}$$

where $\mathbf{1}_{\mathcal{B}_2(I_2)}(r)$ is an indicator function that is equal to one if $r \in \mathcal{B}_2(I_2)$.

In period 1, the entrepreneur makes decisions while considering how their choices will affect the history set $I_2 = \{A_1, o, \omega\}$. If the entrepreneur offers $r_1 \in \mathcal{B}_1(I_1)$, the entrepreneur runs the project, and the history set is updated to $I_2 = \{A_1, 1, \omega\}$, where specific elements of ω depend on the information regime. Conversely, if the entrepreneur offers $r_1 \notin \mathcal{B}_1(I_1)$, the project is not undertaken in period 1, and the history set for period 2 becomes $I_2 = \{A_1, 0, \omega\}$. Based on these observations, the entrepreneur's problem in period 1 is given as

$$\max_{r \in \mathbb{R}_+} \left\{ \begin{array}{c} \mathbf{1}_{\mathcal{B}_1(I_1)}(r) \dfrac{1}{\overline{A}_1} \left\{ \int_{\frac{r}{\theta}}^{\overline{A}_1} (A_1\theta - r + \beta V(\theta, I_2^1)) dA_1 + \int_0^{\frac{r}{\theta}} \beta V_2(\theta, I_2^1) dA_1 \right\} \\ + \left(1 - \mathbf{1}_{\mathcal{B}_1}(r)\right) \beta V_2(\theta, I_2^0) \end{array} \right\} \tag{3}$$

where $\mathbf{1}_{\mathcal{B}_1(I_1)}(r)$ is an indicator function that is equal to one if $r \in \mathcal{B}_1(I_1)$, $I_2^1 = \{A_1, 1, \omega\}$, and $I_2^0 = \{A_1, 0, \omega\}$.

Next, given a belief system μ_t and the public information set I_t in period $t \in \{1, 2\}$, the optimal strategy for lender t is the set of acceptable contracts which is given as

$$\mathcal{B}_t^*(\mu_t, I_t) = \left\{ r_t \in \mathbb{R}_+ : \mu_t\left(r_t, I_t\right) r_t \left(1 - \frac{r_t}{\overline{A}_t \theta_H}\right) + (1 - \mu_t\left(r_t, I_t\right)) r_t \left(1 - \frac{r_t}{\overline{A}_t \theta_L}\right) \geq \gamma \right\}. \tag{4}$$

For a contract to be acceptable, the expected revenue from the entrepreneur's repayment should not be lower than the payoff from investing the investment good in the saving technology that yields γ units of consumption goods in the afternoon with certainty.

4 Equilibrium

In this section, we characterize the equilibrium of the model economy. We adopt Perfect Bayesian Equilibrium (PBE) as our equilibrium concept for the signaling game.

Definition 1 *An equilibrium is a profile of strategies and a belief system,* $\langle \{r_t, D_t\}, \{\mathcal{B}_t\}, \mu_t \rangle_{t\in\{1,2\}}$ *such that for all* $t \in \{1,2\}$*, 1)* $\{r_1, D_1\}$ *solves (3) and* $\{r_2, D_2\}$ *solves (2) for all* $(\theta, I_t) \in \Theta \times \mathbb{I}$*, 2)* $\mathcal{B}_t = \mathcal{B}_t^*(\mu_t, I_t)$*,*[14] *and 3)* $\mu_t(r_t, I_t)$ *is consistent with Bayes' law whenever it is applicable for all* $(r_t, I_t) \in \mathbb{R} \times \mathbb{I}_t$*.*

In the model, the entrepreneur with a different history set $I_2 = \{A_1, o, \omega\}$ could offer different debt contract in period 2. In this case, we refrain from stating that contracts are separating because lender 2 views the entrepreneur with a distinct I_2 as an entirely different borrower. A contract is termed a separating contract only if entrepreneurs of different types, yet with identical I_t offers distinct contracts, including the contract $r_t = 0$, i.e., taking a break in period t.

Specifically, there are two distinct types of separating contracts. First, a contract is deemed separating when the H-type and L-type offer different acceptable contracts to lender t. Second, the H-type and L-type may diverge in their decisions regarding whether to run the project. For example, the H-type might opt not to run a project in period t, offering a contract with $r_t = 0$, while the L-type chooses to proceed with the project, or vice versa. We refer to the first type as a *separating contract without a break* and the second type as a *separating contract with a break.*

As is standard in PBE models, we have multiple equilibria depending on how we construct the lender's belief for off the equilibrium path. In particular, we can have multiple pooling or separating equilibria. When multiple pooling equilibria exist, we choose the pooling equilibrium with the lowest r_t for each period $t \in \{1,2\}$, which we refer to as the least pooling equilibrium. When multiple separating equilibria exist, we

[14] We assume that a lender accepts a contract that makes the lender indifferent between accepting or rejecting the contract, so that the set of acceptable offers is closed.

pick separating equilibrium with the lowest $r_{H,t}$ for each period $t \in \{1, 2\}$, which we call the least separating equilibrium. Here, we focus on $r_{H,t}$ in separating equilibrium because $r_{L,t}$ is the same in any separating equilibrium, which will be manifested later. We call a contract in the least equilibrium of any type the least contract in what follows.

Next, although we focus on the least equilibrium, various types of equilibrium can still exist in the model economy, generating multiple equilibria. Specifically, there can exist an equilibrium where neither type opts to run a project in period 1, despite having access to borrowing the investment good. This equilibrium differs from a market collapse driven by severe adverse selection. Even when the adverse selection problem is not severe, such an equilibrium can arise if both lenders believe any entrepreneur attempting a project in period 1 is of the L-type. We find such a belief system implausible. More importantly, in this equilibrium, real allocations remain the same across all information regimes. Thus, all regimes attain identical welfare levels, offering no policy implications for the optimal information regime. For these reasons, we exclude the characterization of an equilibrium in which both types deliberately choose not to run a project, despite having the option to do so.

Debt contracts with symmetric information Before characterizing equilibrium, suppose that lender t knows the entrepreneur's type in period t. For instance, the entrepreneur's type could be revealed in period 2 through different observable histories. Given that it is optimal for the entrepreneur to default only if he/she has no choice but to default, we obtain the lender t's participation condition as

$$r_{i,t}\left(1 - \frac{r_{i,t}}{\overline{A}_t \theta_i}\right) \geq \gamma \tag{5}$$

for each $t \in \{1, 2\}$ and $i \in \{H, L\}$. Define a function $\hat{r} : [4\gamma, \infty) \to \mathbb{R}$ and a threshold value $\bar{r}_{i,t}$ as

$$\hat{r}(x) = \frac{x - \sqrt{x^2 - 4x\gamma}}{2} \tag{6}$$

$$\bar{r}_{i,t} = \frac{\overline{A}_t \theta_i + \sqrt{\overline{A}_t^2 \theta_i^2 - 4\overline{A}_t \theta_i \gamma}}{2}, \tag{7}$$

for each $t \in \{1, 2\}$ and $i \in \{H, L\}$. Then, the participation constraint (5) holds if and only if $r_{i,t} \in \left[\hat{r}(\overline{A}_t\theta_i), \bar{r}_{i,t}\right]$.

We are focusing on the least equilibrium, so if the entrepreneur chooses to run the project in period t when his/her type is revealed to lender t, the entrepreneur will offer $\hat{r}(\overline{A}_t\theta_i)$ in equilibrium. However, note, from (6), that the entrepreneur of type $i \in \{H, L\}$ can offer $\hat{r}(\overline{A}_t\theta_i)$ only if $\overline{A}_t\theta_i \geq 4\gamma$. Otherwise, the expected return from the project is too low to satisfy the lender's participation constraint (5). Furthermore, because $\hat{r}'(x) < 0$ for all $x \geq 4\gamma$, whenever $\overline{A}_t\theta_L \geq 4\gamma$, we have $\hat{r}(\overline{A}_t\theta_H) < \hat{r}(\overline{A}_t\theta_L)$, indicating that the H-type makes lower repayments than the L-type. This is because the L-type has a higher default risk which must be compensated by the higher repayment.

Because we are interested in the economy with an active credit market, we assume that $\min\{\overline{A}_1, \overline{A}_2\}\theta_H \geq 4\gamma$ so that the H-type can always borrow the investment goods from lender t once his/her type is revealed to lender t. Regarding the productivity of the L-type, we investigate four relevant cases: 1) $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$, 2) $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$, 3) $\overline{A}_1\theta_L \geq 4\gamma > \overline{A}_2\theta_L$, and 4) $\overline{A}_2\theta_L \geq 4\gamma > \overline{A}_1\theta_L$. Note that whenever $\overline{A}_t\theta_L < 4\gamma$, the L-type cannot run a project in period t if their true type is disclosed.

4.1 Equilibrium contract in period 2

We characterize equilibrium by solving the entrepreneur's problem backward, beginning with period 2. The equilibrium outcome in period 2 hinges on whether the entrepreneur's true type is disclosed to lender 2.

If the H-type and L-type entrepreneurs have accumulated distinct history sets I_2, lender 2 can accurately deduce the entrepreneur's true type by observing I_2. In this scenario, the H-type entrepreneur can secure the investment good with the contract $\hat{r}(\overline{A}_2\theta_H)$ with certainty. The L-type entrepreneur can engage in the contract $\hat{r}(\overline{A}_2\theta_L)$ with lender 2 only if $4\gamma \leq \overline{A}_2\theta_L$; otherwise, the L-type cannot borrow the investment good when the type is disclosed in period 2. In summary, we have the following proposition whose proof is omitted.

Proposition 1 *Suppose that the entrepreneur's true type is revealed in period 2 through different history sets I_2. If $4\gamma \leq \overline{A}_2\theta_L$, each type $i \in \{H, L\}$ offers $\hat{r}(\overline{A}_2\theta_i)$; otherwise, only the H-type offers $\hat{r}(\overline{A}_2\theta_H)$.*

The equilibrium outcomes for period 2, when the type is not disclosed, can be derived by solving a standard single-period adverse selection problem in the credit market. We analyze this by categorizing cases based on the relative value of $\overline{A}_2\theta_L$ compared to 4γ.

Proposition 2 *Suppose that $4\gamma \le \overline{A}_2\theta_L$ and the entrepreneur's type has not been revealed to lender 2. Then, both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$, where $\theta_\sigma \equiv \dfrac{\theta_H\theta_L}{(1-\sigma)\theta_H+\sigma\theta_L}$.*

Proof. See Appendix ■

When $4\gamma \le \overline{A}_2\theta_L$, any type of entrepreneur can secure the investment good by offering $\hat{r}(\overline{A}_2\theta_L)$.[15] Although different types can offer distinct contracts in principle, increasing the repayment r only reduces the entrepreneur's expected payoff in period 2, as shown in (1) and (2), since the economy ends in period 2. Thus, contract terms cannot function as a signaling device in period 2 when $4\gamma \le \overline{A}_2\theta_L$, and entrepreneurs will always choose the minimum contract from the pool of acceptable contracts for lender 2. Because the set of acceptable contracts $B_2^*(\mu_2, I_2)$ depends on I_2, both types offer the same contract. The terms of the pooling contract $\hat{r}(\overline{A}_2\theta_\sigma)$ are obtained by substituting the equilibrium consistency condition $\mu_2(r_2, I_2) = \sigma$ into the binding lender's participation constraint (4).

Proposition 3 *Suppose that $\overline{A}_2\theta_L < 4\gamma$ and the entrepreneur's type has not been revealed to lender 2. Then, equilibrium outcomes in period 2 are given as:*

1) *[Market collapse] If $\overline{A}_2\theta_\sigma < 4\gamma$ and $\bar{r}_{2,H} < \overline{A}_2\theta_L$, the entrepreneur of both types cannot borrow the investment good.*
2) *[Pooling] If $4\gamma \le \overline{A}_2\theta_\sigma$, then there exists an equilibrium in which both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2.*
3) *[Separating with a break] If $\overline{A}_2\theta_L \le \bar{r}_{2,H}$, then there exists an equilibrium in which only the H-type offers $\max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$ in period 2.*

Proof. See Appendix ■

If $\overline{A}_2\theta_L < 4\gamma$, the L-type must mimic the H-type to borrow the investment good in period 2. Thus, the L-type can run the project only if both types offer the same pooling contract, $\hat{r}(\overline{A}_2\theta_\sigma)$. Such an equilibrium is feasible only if $4\gamma \le \overline{A}_2\theta_\sigma$; otherwise

[15] Even if lender 2 believes that the entrepreneur who offers $\hat{r}(\overline{A}_2\theta_L)$ is the L-type, i.e., $\mu_2(\hat{r}(\overline{A}_2\theta_L), I_2) = 0$, lender 2 still accepts the offer $\hat{r}(\overline{A}_2\theta_L)$ given that $\overline{A}_2\theta_L \ge 4\gamma$, enabling the entrepreneur to proceed with the project.

lender 2 will reject the offer $\hat{r}(\overline{A}_2\theta_\sigma)$. Meanwhile, the H-type can deter the L-type from mimicking by offering $r \geq \overline{A}_2\theta_L$, as the L-type gains nothing from such a contract in period 2. However, for any contract r offered by the H-type to be accepted by lender 2, it must be $r \in \left[\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}\right]$. Therefore, the H-type can offer this separating contract with a break only if $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$ and will offer $r_{H,2} = \max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$. Finally, if $\overline{A}_2\theta_\sigma < 4\gamma$ and $\bar{r}_{2,H} < \overline{A}_2\theta_L$, the H-type can offer neither a separating contract with a break nor the pooling contract. Thus, the market collapses.

Equilibrium outcomes in period 2, when the type is disclosed, are straightforward as stated in Proposition 1. Therefore, from this point forward, when we refer to equilibrium outcomes in period 2, we will be considering the case where the type is not disclosed unless stated otherwise.

4.2 Equilibrium contract in period 1

We now turn to the equilibrium contract in period 1. While the entrepreneur's decisions in period 2 only affect their expected payoff in that period, decisions made in period 1 have a dual impact: they influence the payoff in period 1 and shape the observable history I_2, which in turn affects the value in period 2. Specifically, the H-type is incentivized to disclose their type to lender 2 by creating a different history I_2 than the L-type. Thus, the information regime could affect the entrepreneur's decisions in period 1. To set the stage for equilibrium characterization, we first outline the terms of a pooling contract in period 1, which are independent of information regimes, while omitting the proof since it parallels that of Proposition 2.

Proposition 4 *If both types offer the same contract in period 1, they will offer $\hat{r}(\overline{A}_1\theta_\sigma)$ under any information regime.*

Before characterizing period 1 contracts, note that when both types offer identical contracts, the transaction history is uninformative, making its disclosure irrelevant. However, default history can be informative because if $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, the H-type repays, while the L-type defaults on the contract $\hat{r}(\overline{A}_1\theta_\sigma)$. Thus, lender 2 can infer the entrepreneur's true type from the default history in this case. Consequently, the disclosure of default history influences real allocations in period 2 when both types offer the pooling contract in period 1.

We now analyze the period 1 equilibrium outcomes for each case based on the relative value of $\overline{A}_1\theta_L$ compared to 4γ. Since the analysis of the equilibrium contract in period 1 offers richer economic insights—while period 2 allocations primarily depend on whether the type is disclosed—we define the equilibrium type based on the contract established in period 1, unless stated otherwise.

4.2.1 Case 1: $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$

When $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$, both lenders are willing to finance the L-type even after their type is revealed. Thus, the H-type cannot deter the L-type from running a project by offering $r \geq \overline{A}_t\theta_L$ in period $t \in \{1, 2\}$.[16] While one type might consider taking a break and the other might not, the following proposition demonstrates that no equilibrium exists where the entrepreneur's decision to run a project in period 1 depends on their type.

Proposition 5 *When* $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$*, there is no equilibrium in which one type of entrepreneur runs the project in period 1 while the other type does not.*

Proof. See Appendix. ■

The intuition behind Proposition 5 is as follows. If the L-type does not run a project in period 1 while the H-type does, the L-type can enhance their payoff by mimicking the H-type's behavior, thereby obtaining project returns in period 1 and subsequently securing more favorable contract terms in period 2. Conversely, if the H-type opts not to run a project in period 1, it is because the expected return from running the project in period 1 is lower compared to the advantage of securing a favorable contract $\hat{r}(\overline{A}_2\theta_H)$ in period 2 by establishing a distinct operational history from the L-type. This implies that the L-type also encounters low returns from a project in period 1, making it optimal for them to take a break as well and secure the same contract in period 2 (see the proof of Proposition 5 for the detail). Thus, an equilibrium where only one type takes a break in period 1 cannot exist.

Based on the results of Proposition 5, when $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$, there can be two distinct types of equilibrium under each regime: 1) a pooling equilibrium, in which both types offer the same contract in period 1, and 2) a separating equilibrium without a break, where each type offers a distinct contract in period 1.

[16] In period 2, for the L-type, offering $\hat{r}(\overline{A}_2\theta_L)$ dominates not running a project, as the economy ends in period 2. Next, in period 1, if the L-type does not run a project when the H-type does with the contract $r \geq \overline{A}_1\theta_L$, the operation history will reveal the L-type's true type. Therefore, for the L-type, offering $\hat{r}(\overline{A}_1\theta_L)$ again dominates not running a project in period 1.

First, in a pooling equilibrium, both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ as described in Proposition 4. In a standard single-period model of PBE, when a pooling contract is acceptable, a pooling equilibrium always exists with an appropriate belief system. However, in our dynamic setting, the analysis becomes more nuanced because the H-type may have an incentive to deviate from the equilibrium path to reveal their true type to lender 2. Specifically, under the full information regime, lender 2 can infer the true type by observing default and transaction histories whenever $A_1 \in \left[\frac{r_1'}{\theta_H}, \frac{r_1'}{\theta_L}\right)$ for any deviating offer $r_1' \neq \hat{r}(\overline{A}_1\theta_\sigma)$.[17] Despite this, the probability that the H-type can reveal their type through deviation in period 1 is too low to be profitable, leading to the following proposition.

Proposition 6 *When* $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$*, an equilibrium in which both types offer the pooling contract* $\hat{r}(\overline{A}_1\theta_\sigma)$ *in period 1 exists under any information regime.*

Proof. See Appendix. ■

We now characterize a separating equilibrium in which the H-type and L-type offer distinct contracts in period 1. The H-type has incentives to offer a different contract to lender 1 than that of the L-type, because by doing so, the H-type may better reveal his/her true type to lender 2. However, this signaling mechanism works only if the transaction history is observable. As a result, the H-type will offer such a separating contract only if lender 2 can observe the transaction history, as demonstrated in the following proposition.

Proposition 7 *An equilibrium in which each type offers a distinct contract in period 1 does not exist if* $r_1 \notin \omega$.

Proof. See Appendix. ■

The result of proposition 7 allows us to narrow our focus to the full-information regime, $\omega = \{r_1, d\}$, and the transaction history regime, $\omega = \{r_1\}$, when characterizing this separating equilibrium. Since $r_1 \in \omega$, the entrepreneur's type is revealed in period 2, as the H-type and L-type offer different contracts in period 1. Moreover, lender 1 can infer the entrepreneur's true type by observing the terms of the offered contract in period 1. Consequently, the L-type offers $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$ in period 1.

[17] In contrast to the full-information regime, the H-type cannot reveal their true type to lender 2 by offering a deviating contract in period 1 under the other three information regimes.

On the other hand, the terms of the period 1 contract for the H-type must be designed to prevent the L-type from mimicking their behavior. Specifically, in this separating equilibrium, an entrepreneur of type $i \in \{H, L\}$ can deceive lender 1 by offering $r_{-i,1}$, where $-i \in \{H, L\} \backslash \{i\}$, in period 1. This deviation can also lead lender 2 to mistakenly believe that the entrepreneur is of type $-i$, unless the credit history reveals the true type.

Specifically, the incentive compatibility constraint for the L-type is given as:

$$\begin{aligned} & u_1\left(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L\right) + \beta u_2\left(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L\right) \\ & \geq u_1\left(r_{H,1} \mid \theta_L\right) + \beta\left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H}\right) u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L\right) \\ & + \beta\left(\frac{r_{H,1}}{\overline{A}_1\theta_L} - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\left\{\begin{array}{c} \mathbf{1}_{\{\omega=\{r_1,d\}\}} u_2\left(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L\right) \\ +(1-\mathbf{1}_{\{\omega=\{r_1,d\}\}}) u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L\right) \end{array}\right\} \end{aligned} \tag{8}$$

where $\mathbf{1}_{\{\omega=\{r_1,d\}\}}$ is an indicator function that takes the value of 1 if $\omega = \{r_1, d\}$ and zero if $\omega = \{r_1\}$. Here, when $A_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$, only the L-type defaults on $r_{H,1}$. Thus, under the full information regime, lender 2 can correctly infer the entrepreneur's true type by observing the default history, even if the L-type offered a deviating contract in period 1. Consequently, the L-type will offer $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Given that the right-hand side of (8) decreases with $r_{H,1}$, the equilibrium contract $r_{H,1}$ is obtained from the binding (8), as outlined in the following proposition.

Proposition 8 *Let r_H^* and r_H^{**} be the values of $r_{H,1}$ that make (8) bind when $\omega = \{r_1, d\}$ and $\omega = \{r_1\}$, respectively. In an equilibrium where each type offers a distinct contract in period 1, $(r_{H,1}, r_{L,1}) = (r_H^*, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1, d\}$, and $(r_{H,1}, r_{L,1}) = (r_H^{**}, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1\}$. Furthermore, $r_H^{**} > r_H^* > \hat{r}(\overline{A}_1\theta_L)$.*

Proof. See Appendix. ∎

In the model, the L-type has an incentive to mimic the H-type. To prevent this, the H-type must offer a higher repayment than the L-type in period 1, i.e., $r_{H,1} > r_{L,1}$. The H-type bears this higher repayment in period 1 to secure more favorable contract terms in period 2 by revealing his/her type through the transaction history. Note that in this separating equilibrium, $r_{H,1} > \hat{r}(\overline{A}_1\theta_L) > \hat{r}(\overline{A}_1\theta_H)$. Consequently, lender 1 earns positive profits from dealing with the H-type, although the entrepreneur has all the bargaining power.

Proposition 8 illustrates that the cost for the H-type to be separated from the L-type is lower under the full-information regime compared to the transaction history regime, i.e., $r_H^{**} > r_H^*$. The intuition for this finding is in line with our earlier observation. If the L-type makes a deviating offer $r_{H,1}$ in period 1 to mimic the H-type, the transaction history is not informative. However, the default history can reveal the entrepreneur's type to lender 2 when $A_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$ since only the L-type defaults on $r_{H,1}$. Thus, the L-type has less incentive to mimic the H-type when $\omega = \{r_1, d\}$ than when $\omega = \{r_1\}$. Consequently, the observability of default history reduces the cost that the H-type incurs to be separated in period 1.

Next, for this separating equilibrium to exist, the H-type must also have no incentive to mimic the L-type. The incentive compatibility constraint for the H-type is given by:

$$\begin{aligned}
&u_1\left(r_{H,1} \mid \theta_H\right) + \beta u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H\right) \\
&\geq u_1\left(\hat{r}(\overline{A}_1\theta_L) \mid \theta_H\right) + \beta\left(1 - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L} + \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right) u_2\left(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H\right) \\
&+ \beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\left\{\begin{array}{c} (1 - \mathbf{1}_{\{\omega=\{r_1\}\}})u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H\right) \\ +\mathbf{1}_{\{\omega=\{r_1\}\}}u_2\left(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H\right) \end{array}\right\}. \qquad (9)
\end{aligned}$$

Here, when $A_1 \in \left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}\right)$, only the H-type can make the repayment on $\hat{r}(\overline{A}_1\theta_L)$. Consequently, lender 2 can correctly infer the entrepreneur's true type, even if the H-type offered the deviating contract $\hat{r}(\overline{A}_1\theta_L)$ in period 1. As a result, the H-type can offer $\hat{r}(\overline{A}_2\theta_H)$ in period 2.

Clearly, any equilibrium offer $r_{H,1}$ must satisfy both incentive compatibility constraints (8) and (9) for this separating equilibrium to exist. Furthermore, $r_{H,1}$ must be such that $r_{H,1} \leq \bar{r}_{1,H}$ for the contract to be accepted by lender 1. Finally, entrepreneurs of both types should not have incentives to take a break in period 1. However, as long as lender 2 believes that an entrepreneur taking a break in period 1 is the L-type off the equilibrium path, opting for a break results only in forfeiting the expected return from running the project in period 1 without improving the contract terms in period 2. Based on this analysis, the following proposition outlines the conditions for the existence of an equilibrium in which each type offers a distinct contract in period 1.

Proposition 9 *Assume that* $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$. *Then, under the full information regime or transaction history regime, an equilibrium in which each type offers a distinct contract in period 1 exists if there is* $r_{H,1} \leq \bar{r}_{1,H}$ *that simultaneously satisfies both (8) and (9).*

Proof. See Appendix. ∎

4.2.2 Case 2: $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$

When $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$, lenders in both periods would refrain from lending to the L-type upon discovering their true type. Consequently, an equilibrium where each type offers a different contract is not feasible. However, if the L-type derives no benefit from mimicking the H-type other than obtaining the investment good in period 1, as is the case in period 2, the H-type can deter the L-type from mimicking by offering $r \geq \overline{A}_1\theta_L$. The key difference in period 1 is that the L-type may still have an incentive to mimic the H-type, even when offered $r \geq \overline{A}_1\theta_L$, in order to conceal their type from lender 2. This incentive depends on the equilibrium outcomes in period 2 when the type is undisclosed.

Proposition 3 shows that when $\overline{A}_2\theta_L < 4\gamma$, three types of equilibrium outcomes can exist in period 2. If the market collapses or only the H-type undertakes the project when the entrepreneur's type is undisclosed in period 2, then mimicking the H-type in period 1 will not enable the L-type to secure the investment good in period 2. Consequently, the L-type has no incentive to mimic the H-type if the H-type offers a contract $r_1' \geq \overline{A}_1\theta_L$ in period 1. This implies that the equilibrium outcomes in period 1 are analogous to the results in Proposition 3 when $\overline{A}_2$ and $\bar{r}_{2,H}$ are replaced with $\overline{A}_1$ and $\bar{r}_{1,H}$, respectively.[18] This leads to the next proposition, the proof of which is omitted.

Proposition 10 *Suppose that* $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$ *and the market collapses or only the H-type offers in period 2 when the type is not disclosed. Then, the equilibrium outcomes in period 1 are given as:*

1) [Market collapse] If $\overline{A}_1\theta_\sigma < 4\gamma$ *and* $\bar{r}_{1,H} < \overline{A}_1\theta_L$, *the market collapses.*

[18] When $\min\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$, if lender t believes the entrepreneur making a deviating offer to be the L-type, the entrepreneur cannot run a project off the equilibrium path, thereby eliminating the incentive to offer a deviating contract.

2) [Pooling] If $4\gamma \leq \overline{A}_1\theta_\sigma$*, then there exists an equilibrium in which both types offer* $\hat{r}(\overline{A}_1\theta_\sigma)$ *in period 1.*

3) [Seperating with a break] If $\overline{A}_1\theta_L \leq \bar{r}_{1,H}$*, then there exists an equilibrium in which only the H-type offers* $\max\left\{\hat{r}(\overline{A}_1\theta_H), \overline{A}_1\theta_L\right\}$ *in period 1.*

Alternatively, if both types offer the pooling contract in period 2 when the type is undisclosed, the L-type can secure the investment good in period 2 by concealing their type from lender 2. Thus, the L-type has an incentive to mimic the H-type, even if the H-type offers a contract $r' \geq \overline{A}_1\theta_L$ in period 1, as otherwise, the operational history will reveal the true type to lender 2. Consequently, it becomes infeasible for the H-type to offer a separating contract with a break in period 1. Therefore, either the market collapses or both types offer the pooling contract, as stated in the following proposition.

Proposition 11 *Suppose that* $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$ *and both types offer the pooling contract in period 2 when the type is not disclosed. Then, the equilibrium outcomes in period 1 are given as:*

1) [Market collapse] If $4\gamma > \overline{A}_1\theta_\sigma$*, the market collapses.*

2) [Pooling] If $4\gamma \leq \overline{A}_1\theta_\sigma$*, then there exists an equilibrium in which both types offer* $\hat{r}(\overline{A}_1\theta_\sigma)$ *in period 1.*

Proof. See Appendix. ■

4.2.3 Case 3: $\overline{A}_1\theta_L < 4\gamma \leq \overline{A}_2\theta_L$

In case 3, the only way for the L-type to borrow the investment good in period 1 is by mimicking the H-type. Note that in period 2, if the type is disclosed through different history sets I_2, the L-type offers $\hat{r}(\overline{A}_2\theta_L)$; otherwise, the L-type offers the pooling contract $\hat{r}(\overline{A}_2\theta_\sigma)$ as shown in Propositions 1 and 2. Since $\hat{r}(\overline{A}_2\theta_L) > \hat{r}(\overline{A}_2\theta_\sigma)$, the L-type always has an incentive to mimic the H-type in period 1, even if the H-type offers a contract $r' \geq \overline{A}_1\theta_L$. Therefore, the H-type cannot deter the L-type from mimicking through a separating contract with a break in period 1, and the equilibrium outcomes in period 1 align with those in Proposition 11: If $4\gamma \leq \overline{A}_1\theta_\sigma$, a pooling equilibrium exists; otherwise, the market collapses.

4.2.4 Case 4: $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$

In case 4, both types can secure the investment good by offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1. While one type might willingly take a break in period 1, this never occurs in equilibrium. Specifically, whenever the H-type runs the project, the L-type is always incentivized to do the same, as taking a break only reveals their true type to lender 2 through the operation history. Conversely, the H-type might consider taking a break to reveal their true type to lender 2, securing better contract terms in period 2. However, when $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$, the expected profit from running the project in period 2 is too low compared to running it in period 1, to incentivize the H-type to adopt this strategy (see the proof of Proposition 12 for the detail). Consequently, neither type opts to take a break in period 1, as stated in the next proposition, similar to Case 1.

Proposition 12 *When $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$, no equilibrium exists in which only one type of entrepreneur runs the project in period 1.*

Proof. See Appendix. ■

Given the results of Proposition 12, we now focus on an equilibrium where both types offer a pooling contract (pooling equilibrium) and an equilibrium where each type offers a distinct contract (separating equilibrium without a break).

In a pooling equilibrium, both types offer the pooling contract $\hat{r}(\overline{A}_1\theta_\sigma)$, which is acceptable to lender 1 given the consistent belief $\mu_1\left(\hat{r}(\overline{A}_1\theta_\sigma), I_1\right) = \sigma$ when $4\gamma \leq \overline{A}_1\theta_L$, as in case 1. For this pooling equilibrium to exist, neither type should have incentives to deviate by offering a contract $r' \neq \hat{r}(\overline{A}_1\theta_\sigma)$ in period 1. However, due to the dynamic nature of our model, the H-type might be tempted to deviate, even at the risk of being perceived as the L-type by lender 1, to reveal their true type in period 2. Based on these considerations, the next proposition describes the existence of a pooling equilibrium in case 4.

Proposition 13 *Assume that $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$. If the market collapses in period 2 when the type is not disclosed, then there exists $A_2^* \in \left(\frac{4\gamma}{\theta_H}, \frac{4\gamma}{\theta_L}\right]$ such that for all $\overline{A}_2 > A_2^*$, a pooling equilibrium, where both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ in period 1, cannot exist under the full information regime while it exists under the other information regimes. Otherwise, a pooling equilibrium exists under any information regime.*

Proof. See Appendix. ■

In equilibrium where both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$, the H-type can better reveal their type to lender 2 by offering a deviating contract $r' > \hat{r}(\overline{A}_1\theta_\sigma)$ under the full information regime, as lender 2 can infer that only the H-type can repay r' when $A_1 \in \left[\frac{r'_1}{\theta_H}, \frac{r'_1}{\theta_L}\right)$.[19] However, if the H-type can secure the investment good in period 2, even when their type is undisclosed—as in Case 1—this deviation does not generate sufficient net profit to incentivize the H-type to deviate.

The key difference in Case 4, compared to Case 1, is that the market can collapse, as demonstrated in Proposition 3. In such scenarios, the H-type gains more by revealing their type to lender 2, making deviation more attractive. In particular, an increase in $\overline{A}_2$ raises the return from running the project in period 2, thus increasing the H-type's incentive to deviate. Consequently, when the market collapses in period 2, the H-type lacks the incentive to deviate, thereby establishing a pooling equilibrium where both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ in period 1, only if $\overline{A}_2$ is sufficiently low, such that $\overline{A}_2 \leq A_2^*$.

Next, since $4\gamma \leq \overline{A}_1\theta_L$ in case 4, the entrepreneur of any type can always borrow the investment good with the contract $\hat{r}(\overline{A}_1\theta_L)$ in period 1. Consequently, an equilibrium where each type offers a distinct separating contract in period 1 can potentially exist. This separating equilibrium is feasible only if lender 2 can observe the transaction history, as illustrated in Proposition 7, for the same reasons as in case 1.

In this equilibrium, the L-type offers $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$ in period 1 because their type is disclosed through the terms of the contract. The true type is also revealed in period 2 through the transaction history, preventing the L-type from borrowing the investment goods. Consequently, only the H-type runs the project with the contract $\hat{r}(\overline{A}_2\theta_H)$. Therefore, the incentive compatibility constraint for the L-type in period 1 is given as

$$u_1\left(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L\right) \geq u_1\left(r_{H,1} \mid \theta_L\right) + \beta\left\{1 - \mathbf{1}_{\{\omega=\{r_1,d\}\}}\left(\frac{r_{H,1}}{\overline{A}_1\theta_L} - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\right\} u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L\right), \tag{10}$$

and the terms of contract $r_{H,1}$ for the H-type in period 1 are determined by the binding constraint (10).

Similar to case 1, the H-type can offer a separating contract without a break at a lower cost under the full information regime compared to the transaction history

[19] Clearly, this strategy does not work under other information regimes, and it can only lead lender 1 to believe the deviating entrepreneur is the L-type.

regime. This is because, when $A_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$, the default history reveals the true type of the deviating L-type in period 2, thereby reducing the L-type's incentive to mimic in period 1. In summary, we have the following proposition, whose proof is similar to that of Proposition 8, and is therefore omitted.

Proposition 14 *Suppose that $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$, and let $\tilde{r}_H^*$ and $\tilde{r}_H^{**}$ be the values of $r_{H,1}$ that make (10) bind when $\omega = \{r_1, d\}$ and $\omega = \{r_1\}$, respectively. Then, in an equilibrium where each type offers a distinct contract in period 1, $(r_{H,1}, r_{L,1}) = (\tilde{r}_H^*, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1, d\}$ and $(r_{H,1}, r_{L,1}) = (\tilde{r}_H^{**}, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1\}$. Furthermore, $\tilde{r}_H^{**} > \tilde{r}_H^* > \hat{r}(\overline{A}_1\theta_L)$.*

We now examine the existence of this separating equilibrium without a break in period 1 in Case 4. For this equilibrium to exist, the separating contract, $\tilde{r}_H^*$ or $\tilde{r}_H^{**}$ depending on the information regime, must be weakly lower than $\bar{r}_{1,H}$; otherwise, lender 1 will not accept it. Additionally, the H-type must have no incentive to mimic the L-type for this equilibrium to hold. In Case 4, if the H-type mimics the L-type in period 1, they cannot borrow the investment good in period 2 unless the default history reveals their true type. Consequently, the H-type has less incentive to mimic the L-type in Case 4 compared to Case 1. In particular, any equilibrium contract $r_{H,1}$ that satisfies the L-type's incentive constraint (10) with equality also satisfies the H-type's incentive compatibility constraint (as detailed in the proof of Proposition 15). This leads to the next proposition.

Proposition 15 *Assume that $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$. Then, under the full information regime (transaction history regime), an equilibrium in which each type offers a distinct contract in period 1 exists if $\tilde{r}_H^* \leq \bar{r}_{1,H}$ ($\tilde{r}_H^{**} \leq \bar{r}_{1,H}$).*

Proof. See Appendix ■

4.3 Credit history disclosure on loan rates

We close this section by exploring how the disclosure of default history and transaction history influences loan rates. To isolate the effects of aggregate productivity, we assume $\overline{A}_1 = \overline{A}_2$ and $\overline{A}_1\theta_L \geq 4\gamma$, concentrating on Case 1, which offers the richest implications for loan rate dynamics and dispersion.

In traditional credit markets, lenders typically access borrowers' default histories via credit reports but cannot observe transaction histories—paralleling the default history regime in our model. Under this regime, both types offer pooling contracts in period 1. In period 2, if the L-type defaults while the H-type does not in period 1, each type $i \in \{H, L\}$ makes a contract of $\hat{r}(\overline{A}_2\theta_i)$. Otherwise, both types borrow under a pooling contract $\hat{r}(\overline{A}_2\theta_\sigma)$. Since $\hat{r}(\overline{A}_2\theta_H) < \hat{r}(\overline{A}_2\theta_\sigma) < \hat{r}(\overline{A}_2\theta_L)$, a default history of $d = 1$ increases borrowing costs in period 2, consistent with empirical findings (e.g., Peri and Rachedi (2019)). Moreover, this result implies that borrowing costs generally decline for borrowers without default experiences, aligning with evidence from Berger and Udell (1995) and Bharath et al. (2011).

The economic effects of disclosing transaction histories have received little academic attention, as such histories are typically unobservable in traditional credit markets. However, advancements in DeFi have introduced new possibilities for analyzing these effects. Platforms like Aave publicly record all transaction details, including contract terms, while platforms like Maple Finance and TrueFi limit disclosure to specific borrower information, excluding transaction histories.

In our model, when transaction histories are observable, a separating equilibrium emerges where both types offer distinct contracts in both periods. This contrasts with the default history regime, where distinct contracts arise only in period 2, following the revelation of type information through default histories. Accordingly, the model predicts that transaction history disclosure increases loan rate dispersion, even among first-time borrowers, by facilitating separating contracts. While empirical testing of this prediction could deepen our understanding, it demands tailored strategies to accommodate the unique characteristics of each DeFi platform, which we leave for future research.

5 Welfare Analysis

In this section, we analyze welfare across different information regimes and identify the optimal regime for information disclosure. Before delving into welfare, note that when $4\gamma \leq \overline{A}_1\theta_L$, an equilibrium where each type offers a distinct contract in period 1 (separating equilibrium without a break) can coexist with an equilibrium where both types offer the pooling contract (pooling equilibrium) in period 1, provided transaction histories are observable. Since our focus is on determining the optimal information regime, we assume the existence of a separating equilibrium when both equilibria coexist. This implies that the government can ensure the existence of a pooling equilibrium by withholding transaction histories.

5.1 Welfare measure

We use the sum of utility across agents and periods in equilibrium as our welfare measure. Specifically, define welfare in each period as follows:

$$W_t = \sigma \left\{ u_t(r_{t,H} \mid \theta_H) + \left(1 - \frac{r_{t,H}}{\overline{A}_t \theta_H}\right) r_{t,H} \right\} + (1-\sigma) \left\{ u_t(r_{t,L} \mid \theta_L) + \left(1 - \frac{r_{t,L}}{\overline{A}_t \theta_L}\right) r_{t,L} \right\} - \gamma. \tag{11}$$

Then, our welfare measure is given as

$$W = W_1 + \beta W_2. \tag{12}$$

Note, from (6), that $\hat{r}(\cdot)$ is a convexly decreasing function. Thus, in each period $t \in \{1, 2\}$, the liquidation cost from the pooling contract $\hat{r}(\overline{A}_t \theta_\sigma)$ is lower than that from the average liquidation cost from $\hat{r}(\overline{A}_t \theta_i)$ for each $i \in \{H, L\}$. This leads to the next lemma which provides a useful intermediate step for the welfare analysis.

Lemma 1 *Suppose that $\overline{A}_t \theta_L \geq 4\gamma$. Then, W_t is higher when $r_{t,H} = r_{t,L} = \hat{r}(\overline{A}_t \theta_\sigma)$ than when $r_{t,i} = \hat{r}(\overline{A}_t \theta_i)$ for each $i \in \{H, L\}$.*

Proof. See Appendix ∎

DeFi platform profits and welfare Decentralized Finance (DeFi) marks a transformative shift in the financial sector by leveraging blockchain technology to establish an open, transparent, and permissionless financial system. A key feature of blockchain, the foundation of DeFi, is its ability to record every transaction on a public ledger, ensuring transparency and enabling users to verify the integrity of financial processes. When crafting a DeFi white paper, developers can specify which transaction details will be recorded on the blockchain and made public.[20] Within

[20] A white paper is a comprehensive document that outlines the concept, technical details, and the proposed functionality of a DeFi platform. White papers are often published by the developers of a DeFi platform to explain their vision, the problem they aim to solve, and how their solution works.

our model, this includes decisions on disclosing credit history information—such as defaults and transaction histories—on the blockchain.

To analyze the DeFi platform's problem of information disclosure within our framework, we adapt the model as follows. We assume a continuum of entrepreneurs and lenders, each with unit mass, operating over two periods with a discount factor $\beta \in (0,1)$. A proportion $\sigma \in (0,1)$ of entrepreneurs have high productivity θ_H, while the remaining entrepreneurs have low productivity θ_L. The productivity of each entrepreneur is private information.

Additionally, there exists a monopoly platform that facilitates a decentralized credit market in the morning in each period. To participate, entrepreneurs and lenders must pay a one-time fee at the start of period 1, granting access to the platform for both periods. After paying the fee, borrowers and lenders are randomly matched, and in each bilateral meeting, the borrower makes an offer to the lender.

Since entrepreneurs and lenders are randomly matched each period, the probability of pairing the same entrepreneur and lender in both periods is zero. Therefore, the only information available to lenders for evaluating an entrepreneur's type in period 2 is the public information set $I_2 = \{A_1, o, \omega\}$, where specific components of the credit history ω are predetermined by the platform. Consequently, the optimal strategies for entrepreneurs and lenders in the modified model are identical to those in our model with a single entrepreneur and two lenders.

The platform will choose the information regime, determining which histories to disclose in period 2, and set fees to maximize its profit by extracting all trade surpluses from entrepreneurs and lenders. Consequently, the platform's problem reduces to identifying the information regime that maximizes the total trade surplus, which corresponds exactly to the welfare measure defined in (12). Thus, the optimal information regime we will examine is the one that maximizes the platform's profit in the modified model.

5.2 Optimal information regime

We now explore the optimal information regime. As in the equilibrium analysis in section 4, we assess welfare based on the L-type's relative productivity in each period. In what follows, W_ω denotes welfare under information regime ω. Specifically, $W_\varnothing$, $W_{\{r_1,d\}}$, $W_{\{d\}}$, and $W_{\{r_1\}}$ represent welfare under the no-information, full-information, default history, and transaction history regimes, respectively.

5.2.1 Case 1: $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$

When $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$, both types undertake the project in each period across all equilibria and information regimes. Moreover, it is socially desirable for the L-type to run the project in both periods. Therefore, the optimal information regime is the one that minimizes the liquidation costs from defaults in both periods.

Given that $\overline{A}_2\theta_L \geq 4\gamma$, if the type is not disclosed in period 2, both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$; otherwise, each type $i \in \{H, L\}$ offers $\hat{r}(\overline{A}_2\theta_i)$, as demonstrated in Propositions 1 and 2. Then, by the results of Lemma 1, W_2 is higher when the type is not disclosed. Consequently, W_2 is higher under a pooling equilibrium, where both types offer the same contract in period 1, than under a separating equilibrium without a break, where distinct contracts are offered in period 1, because the type is fully revealed in period 2 in the latter.

The same rationale applies to W_1 in period 1. Given that $\overline{A}_1\theta_L \geq 4\gamma$, W_1 is higher when both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ than when each type $i \in \{H, L\}$ offers $\hat{r}(\overline{A}_1\theta_i)$, as shown by Lemma 1. Additionally, in a separating equilibrium without a break, the H-type's contract $r_{H,1}$ in period 1 is higher than $\hat{r}(\overline{A}_1\theta_H)$, leading to a greater default probability and increased liquidation costs. Taken together, welfare is higher when both types offer the pooling contract in period 1 rather than when each type offers a distinct contract.

Consequently, withholding transaction history is advantageous, as its disclosure can only lead to a separating equilibrium without a break. Now, assuming $r_1 \notin \omega$, both types will offer the pooling contract in period 1 in equilibrium. Then, disclosing the default history reduces W_2 without affecting W_1, as it reveals the true type when A_1 is in an intermediate range. These findings lead to the conclusion that the no-information regime, i.e., withholding both default and transaction histories, is optimal in Case 1.[21] This is re-emphasized in the next proposition, whose proof is omitted.

Proposition 16 *When* $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$*, the no-information regime is the optimal information regime.*

[21] Andolfatto et al. (2014) and Andolfatto and Martin (2013) explore the macroeconomic effects of asset quality disclosure. Their models show that asset quality affects trade volumes—higher dividends increase trade, while lower dividends reduce it—making nondisclosure optimal for facilitating consumption smoothing across states.

5.2.2 Case 2: $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$

In Case 2, the outcome in period 1 depends on the economic environment: the market may collapse, both types may offer a pooling contract, or only the H-type may offer a separating contract with a break, as detailed in Propositions 10 and 11. If the market collapses in period 1, the credit history contains no information, i.e., $\omega = \varnothing$. If only the H-type undertakes the project in period 1, the operation history reveals the entrepreneur's type to lender 2. Therefore, in both scenarios, the information regime becomes irrelevant.

Now, suppose both types offer a pooling contract in period 1. In this case, disclosing the default history matters, while the transaction history does not, because the default history reveals the true type in period 2 if $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Then, if only the H-type offers in period 2 when the type is not disclosed, revealing the default history is optimal, as the H-type can borrow at a weakly lower cost when their type is revealed.[22] Similarly, if the market collapses in period 2, disclosing the default history is optimal, enabling the H-type to run the project. Finally, if both types offer a pooling contract in period 2, type disclosure ensures that only the H-type can run the project with the contract $\hat{r}(\overline{A}_2\theta_H)$; without disclosure, both types run the project with the contract $\hat{r}(\overline{A}_2\theta_\sigma)$. Thus, the welfare difference between the default history regime and the no-information regime is given as

$$W_{\{d\}} - W_\varnothing = \beta\left[\sigma\left\{u_2\left(\hat{r}(\overline{A}\theta_H) \mid \theta_H\right) - u_2\left(\hat{r}(\overline{A}\theta_\sigma) \mid \theta_H\right)\right\} - (1-\sigma)u_2\left(\hat{r}(\overline{A}\theta_\sigma) \mid \theta_L\right)\right]. \tag{13}$$

Then, it is optimal to disclose the default history if the term in (13) is positive, and vice versa.

To scrutinize the term $W_{\{d\}} - W_\varnothing$ in (13), we express the necessary conditions for both types to offer a pooling contract in period 2 in terms of σ. Specifically, note that when $\overline{A}_2\theta_L < 4\gamma \le \overline{A}_2\theta_H$, there exists a unique $\sigma_1 \in (0,1]$ such that $4\gamma = \overline{A}_2\theta_\sigma$ when $\sigma = \sigma_1$. Next, it can be verified that if $\overline{A}_2\theta_L > 2\gamma$, $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ for all $\sigma \in [0,1]$, while if $\overline{A}_2\theta_L \le 2\gamma$, there exists a unique $\sigma_2 \in [\sigma_1, 1]$ such that $\hat{r}(\overline{A}_2\theta_\sigma) = \overline{A}_2\theta_L$ when

[22] In a separating equilibrium with a break, the H-type offers $\max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$, while when the type is disclosed the H-type offers $\hat{r}(\overline{A}_2\theta_H)$. Thus, the average liquidation cost is weakly lower when the type is disclosed.

$\sigma = \sigma_2$.[23] Now define the threshold value of σ as

$$\sigma^* = \begin{cases} \sigma_1 & \text{if } 2\gamma < \overline{A}_2\theta_L < 4\gamma \\ \sigma_2 & \text{if } \overline{A}_2\theta_L \leq 2\gamma. \end{cases} \tag{14}$$

Then, for all $\sigma > \sigma^*$, there exists an equilibrium in which both types offer a pooling contract because $\overline{A}_2\theta_\sigma$ increases with σ.[24] Utilizing the definition of σ^*, the following proposition examines the optimal information regime for Case 2, specifically when both types offer pooling contracts in both periods if the type is disclosed.

Proposition 17 *Suppose that* $\max\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$ *and an equilibrium exists where both types offer the pooling contract in both periods when the entrepreneur's type is not disclosed. Then, there is* $\theta_L^* \in \left(\frac{2\gamma}{\overline{A}_2}, \frac{4\gamma}{\overline{A}_2}\right)$ *such that if* $\theta_L \leq \theta_L^*$*, there exist* $\sigma^{**} \in (\sigma^*, 1]$ *such that for all* $\sigma \in (\sigma^*, \sigma^{**})$, $W_{\{d\}} > W_\varnothing$ *and for all* $\sigma \in [\sigma^{**}, 1]$, $W_\varnothing \geq W_{\{d\}}$.

Proof. See Appendix. ■

When $\min\{\overline{A}_1, \overline{A}_2\}\theta_L < 4\gamma$, the return from the L-type's project is insufficient to induce lender 2 to lend the investment good to the L-type. Nevertheless, this does not necessarily signify social inefficiency in allowing the L-type to run the project, as the L-type can still achieve a positive return by forming a pooling contract with lender 2. The combined expected payoffs for lender 2 and the L-type from entering into a pooling contract can exceed γ.

However, if θ_L is sufficiently low, the L-type is highly likely to default on the pooling contract $\hat{r}(\overline{A}\theta_\sigma)$, significantly reducing the L-type's expected payoff $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L\right)$. Specifically, as $\overline{A}_2\theta_L$ converges to $\hat{r}(\overline{A}_2\theta_\sigma)$, $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L\right)$ approaches zero, making $W_{\{d\}} > W_\varnothing$ as illustrated in (13). Similarly, as σ falls, $\hat{r}(\overline{A}_2\theta_\sigma)$ increases, further reducing $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L\right)$ while widening the gap $u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H\right) - u_2\left(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H\right)$ in (13). Thus, if θ_L and σ are sufficiently low, it is socially optimal for the L-type not to run the project, making the default history regime optimal.

To obtain further policy implications, we conduct a numerical analysis. We set the parameters as follows: $\gamma = 1.2$, $\theta_H = 1.5$, $\theta_L = 1$, $\sigma \in [0, 1]$, $\overline{A}_1 = 4$, and

[23] Note, from (6), that $\hat{r}(\cdot)$ is a decreasing function and $\hat{r}(4\gamma) = 2\gamma$. Hence, $4\gamma \leq \overline{A}_2\theta_\sigma$ implies $\hat{r}(\overline{A}_2\theta_\sigma) \leq 2\gamma$. This, in turn, implies $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ when $\overline{A}_2\theta_L > 2\gamma$. On the other hand, when $\overline{A}_2\theta_L \leq 2\gamma$, $\sigma_2 \in [0, 1]$ is well defined by the condition that $\hat{r}(\overline{A}_2\theta_\sigma) = \overline{A}_2\theta_L$ with $\sigma = \sigma_2$. Furthermore, $4\gamma \leq \overline{A}_2\theta_\sigma$ holds when $\sigma = \sigma_2$ in this case. Thus, it must be that $\sigma_2 \geq \sigma_1$.

[24] Note that by construction of σ^*, for all $\sigma > \sigma^*$, $4\gamma < \overline{A}\theta_\sigma$ and $\hat{r}(\overline{A}\theta_\sigma) < \overline{A}\theta_L$. In the knife-edge case, in which $4\gamma \leq \overline{A}\theta_\sigma$ and $\hat{r}(\overline{A}\theta_\sigma) < \overline{A}\theta_L$ when $\sigma = \sigma^*$, pooling equilibrium can also exist with $\sigma = \sigma^*$.

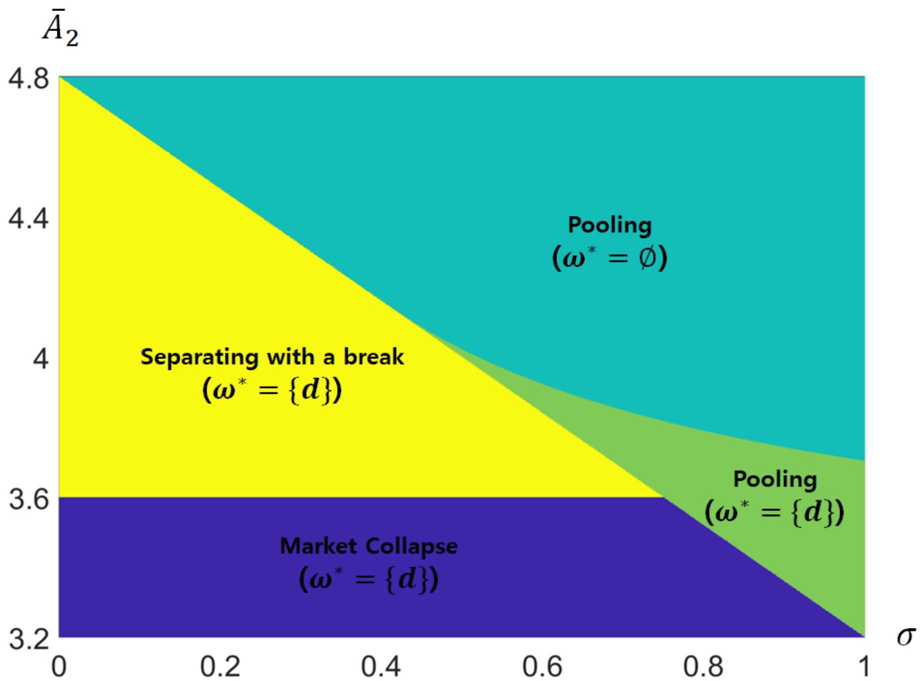

Figure 1: Typology of equilibria in period 2 in $(\sigma, \overline{A}_2)$-space and the optimal information regime in Case 2

$\overline{A}_2 \in [3.2, 4.8]$, ensuring that the condition $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_H$ is satisfied. Based on our parameter choices, Figure 1 illustrates the contract types, including the market collapse as a type of contract, in period 2 when the type is not disclosed, along with the optimal information regime, denoted by ω^*. For instance, a region with "pooling $(\omega^* = \{d\})$" implies that in period 2, if the type is undisclosed, both types offer the pooling contract, and welfare is higher when the type is disclosed, making it optimal to reveal the default history.

Figure 1 indicates that disclosing the default history is generally optimal when $\overline{A}_2$ and σ are sufficiently low. This is intuitive because as $\overline{A}_2$ decreases, the expected return from the L-type's project diminishes, making the L-type less socially desirable for running their project. Additionally, a decrease in σ exacerbates the adverse selection problem. Consequently, when $\overline{A}_2$ and σ are low, the social benefit of revealing the entrepreneur's true type through default history in period 2 becomes substantial.

5.2.3 Case 3: $\overline{A}_1\theta_L < 4\gamma \leq \overline{A}_2\theta_L$

In Case 3, according to Proposition 11, an equilibrium exists where either the market collapses or both types offer a pooling contract in period 1. Since the information regime is irrelevant if the market collapses, we assume $4\gamma \leq \overline{A}_1\theta_\sigma$, ensuring that both types offer a pooling contract in period 1.

In this context, disclosing the transaction history is irrelevant because both types offer the same contract in period 1. However, the default history reveals the true type when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Despite this, given that $4\gamma \leq \overline{A}_2\theta_L$, disclosing the default history is suboptimal, as shown in Lemma 1. Therefore, both the no-information and transaction history regimes, which withhold the default history, are optimal. This result is formally stated in the following proposition, with the proof omitted.

Proposition 18 *When $\overline{A}_1\theta_L < 4\gamma \leq \overline{A}_2\theta_L$, withholding the default history is optimal. Consequently, both the no-information and transaction history regimes are optimal.*

5.2.4 Case 4: $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_1\theta_L$

The equilibrium outcomes for period 2 in Case 4 are equivalent to those in Case 2. However, unlike Case 2, Case 4 permits an equilibrium where each type offers distinct contracts in period 1, provided the transaction history is observable. As noted in Proposition 14, the H-type can offer a lower separating contract under the full information regime than under the transaction history regime, leading to lower average liquidation costs and higher welfare. Thus, in Case 4, the full information regime dominates the transaction history regime.

Accordingly, we compare welfare in a separating equilibrium under full information, a pooling equilibrium under the no-information regime, and a pooling equilibrium under the default history regime to identify the optimal information regime in Case 4.[25] Specifically, we perform a numerical analysis, employing the same parameter values as those used in case 2, but with $\overline{A}_1 \in \{8, 12\}$, ensuring that $4\gamma \leq \overline{A}_1\theta_L$. The left and right panels of Figure 2 depict the contract types, including the market collapse, when the type is not disclosed, along with the corresponding optimal

[25] The equilibrium type is defined according to the type of contract in period 1, as stated earlier.

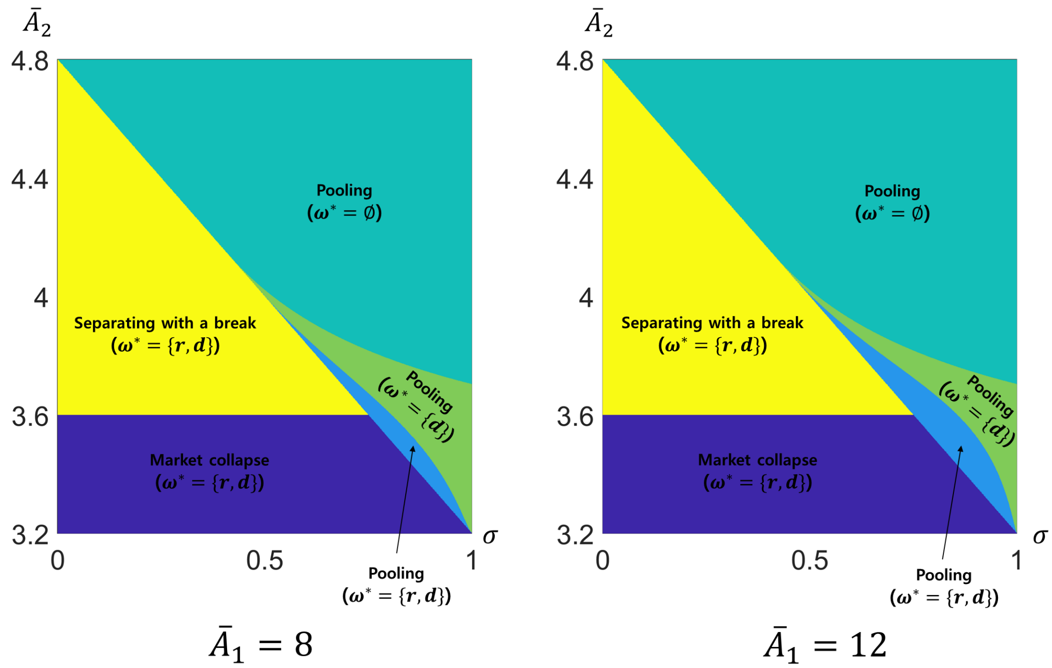

Figure 2: Typology of equilibria in period 2 in $(\sigma, \overline{A}_2)$-space and the optimal information regime in Case 4

information regime, ω^*, for $\overline{A}_1 = 8$ and $\overline{A}_1 = 12$, respectively.

A decrease in $\overline{A}_2$ and σ aggravates the consequences of the adverse selection problem when the type is undisclosed in period 2. Thus, when $\overline{A}_2$ or σ is sufficiently low, disclosing more information about credit history is optimal, as it helps mitigate the adverse selection problem, similar to Case 2. However, unlike Case 2, where disclosing transaction history is irrelevant, here it can support a separating equilibrium where each type offers distinct contracts in period 1. When such a separating equilibrium exists, the transaction history reveals the entrepreneur's type with certainty, making it more effective than default history in unveiling the entrepreneur's type.

However, disclosing the transaction history is more costly than disclosing the default history. When $4\gamma \leq \overline{A}_1\theta_L$, the welfare measure W_1 in period 1 is higher when both types offer a pooling contract than when each type offers distinct contracts by the results of Lemma 1. Thus, the full information regime is optimal only if the adverse selection problem is severe, with sufficiently low $\overline{A}_2$ and/or σ. As $\overline{A}_2$ and σ rise, the social benefit of revealing the entrepreneur's type in period 2 diminishes, making the default history regime optimal despite its imperfect disclosure. Finally, if $\overline{A}_2$ and σ are sufficiently high, it becomes optimal to withhold disclosure of both histories, making the no information regime optimal.

Another result shown in Figure 2 is that the full information regime is more likely to dominate the default history regime as $\overline{A}_1$ increases. The intuition behind this is as follows. Under the full information regime, each type offers distinct contracts in period 1, and the transaction history always reveals the type in period 2. In contrast, under the default history regime, both types offer the same contract in period 1, and the type is disclosed in period 2 only if $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Thus, the probability of type disclosure under the default history regime is $\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)$, which decreases as $\overline{A}_1$ increases. Consequently, the default history regime becomes less effective at disclosing the type compared to the full information regime as $\overline{A}_1$ rises.

6 Conclusion

In this paper, we have developed a dynamic model of debt contracts under adverse selection. An entrepreneur borrows investment goods from lenders to finance a project, with the project's return depending on both aggregate productivity and the entrepreneur's private productivity. Lenders assess the entrepreneur's productivity using their credit history, including transaction and/or default history, alongside past aggregate productivity. We have analyzed how disclosing these histories influences real allocations, how the effects vary with current and future economic conditions, and investigated the optimal information regime.

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Appendix: Proof

Proof of proposition 2. Because the entrepreneur's type has not been revealed to lender 2, lender 2 expects the entrepreneur to be the H-type with probability σ and the L-type with probability $1-\sigma$. Let r be the contract that both types offer. Then the H-type repays with probability $1-\frac{r}{\overline{A}_2\theta_H}$ and the L-type repays with probability $1-\frac{r}{\overline{A}_2\theta_L}$. As a result, the lender's expected payoff is

$$\sigma r\left(1-\frac{r}{\overline{A}_2\theta_H}\right)+(1-\sigma)r\left(1-\frac{r}{\overline{A}_2\theta_L}\right)=r\left(1-\frac{r}{\overline{A}_2\theta_\sigma}\right),$$

where $\theta_\sigma \equiv \dfrac{\theta_H \theta_L}{(1-\sigma)\theta_H + \sigma\theta_L}$. Finally, the smallest r that satisfies $r\left(1 - \dfrac{r}{\overline{A}_2\theta_\sigma}\right) \geq \gamma$ is $\hat{r}(\overline{A}_2\theta_\sigma)$. ■

Proof of proposition 3. We first introduce and prove the following claim, which provides a useful intermediate step.

Claim 1 *There exists r such that $r\left(1 - \dfrac{r}{A\theta}\right) \geq \gamma$ if and only if $A\theta \geq 4\gamma$.*

Proof. $r\left(1 - \dfrac{r}{A\theta}\right) \geq \gamma$ is equivalent to $r^2 - A\theta r + A\theta\gamma \leq 0$. Thus, there exists r such that $r^2 - A\theta r + A\theta \leq 0$ if and only if $A\theta \geq 4\gamma$. ■

We now prove the proposition. Given that $\overline{A}_2\theta_L < 4\gamma$, the L-type cannot offer an acceptable contract if their type is disclosed in period 2. Thus, the only way that the L-type to borrow is when their type is not disclosed to mimic the H-type.

If $\overline{A}_2\theta_\sigma \geq 4\gamma$, then, by claim 1, there exists a pooling offer, $\hat{r}(\overline{A}_2\theta_\sigma)$, that the lender accepts in period 2 with the consistent belief. With the support of the lender's belief that any deviation from the pooling offer is of the L-type, there exists a pooling equilibrium that both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2. On the other hand, if $\overline{A}_2\theta_\sigma < 4\gamma$, there is no acceptable pooling contract in period 2 by the results of claim 1. Thus, an equilibrium where both types offer the pooling contract cannot exist. However, an equilibrium where only the H-type offers a contract that deters the L-type from mimicking may exist, and we focus on the existence of such equilibrium.

First, suppose that $\bar{r}_{2,H} < \overline{A}_2\theta_L$. Note that any contract $r_{H,2}$ offered by the H-type must be $r_{H,2} \in [\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}]$ because a contract will be rejected by lender 2 otherwise. Consider any contract $r_{H,2} \in [\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}]$. The expected payoff of the L-type from mimicking this contract is $\int_{A_2 \geq \frac{r_{H,2}}{\theta_L}} \max\{0, A_2\theta_L - r_{H,2}\}$, which is positive given that $\bar{r}_{2,H} < \overline{A}_2\theta_L$.

Second, suppose $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$. Note that for any contract $r_{H,2} < \overline{A}_2\theta_L$, the L-type has an incentive to mimic the H-type. However, for any contract $r_{H,2} \geq \overline{A}_2\theta_L$, the L-type has no incentive to mimic since their payoff is zero. Consequently, the H-type can deter the L-type from mimicking by offering a contract $r_{H,2} \geq \overline{A}_2\theta_L$. Because we focus on the least equilibrium and it must be that $r_{H,2} \in [\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}]$ for a contract to be accepted, we obtain $r_{H,2} = \max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$ as an equilibrium seperating contract with a break in period 2. Consequently, an equilibrium where only the H-type offers in period 2 can exist only if $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$.

Finally, if $\bar{r}_{2,H} < \overline{A}_2\theta_L$ and $\overline{A}_2\theta_\sigma < 4\gamma$, the H-type cannot offer either a separating contract with a break or a pooling contract. Consequently, the market collapses. ■

Proof of proposition 5. To bolster the credibility of this equilibrium, we introduce the assumption that any deviation from the established equilibrium path results in the worst belief regarding the entrepreneur's type unless the type is revealed. We first introduce and prove the following claims.

Claim 2 *In each period t and $q \in \{1, 0, \sigma\}$, $r = \hat{r}(\overline{A}_t\theta_q)$ is the lowest r that satisfies*

$$qr\left(1 - \frac{r}{\overline{A}_t\theta_H}\right) + (1-q)r\left(1 - \frac{r}{\overline{A}_t\theta_L}\right) \geq \gamma,$$

where $\theta_1 = \theta_H$ and $\theta_0 = \theta_L$.

Proof of claim 2. By rearranging the inequality, we get $r\left(1 - \frac{r}{\overline{A}_t\theta_q}\right) \geq \gamma$. Therefore, according to (5) and (6), $r = \hat{r}(\overline{A}_t\theta_q)$ is the lowest r that satisfies $r\left(1 - \frac{r}{\overline{A}_t\theta_q}\right) \geq \gamma$. ■

Claim 3 *If $\overline{A}_2\theta_L \geq 4\gamma$, then for any $\theta_H \geq \theta_1 > \theta_2 \geq \theta_L$,*

$$\left(1 - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2\theta_L}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2\theta_L}\right)^2 > \left(1 - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2\theta_H}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2\theta_H}\right)^2.$$

Proof or claim 3. Because $\hat{r}(\overline{A}_2\theta_2) > \hat{r}(\overline{A}_2\theta_1)$, it suffices to show that $\left(1 - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2\theta}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2\theta}\right)^2$ decreases in θ. Noting that

$$\left(1 - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2\theta}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2\theta}\right)^2 = \left(\frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2}\right)\left[\frac{2}{\theta} - \left(\frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} + \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2}\right)\frac{1}{\theta^2}\right],$$

we get

$$\begin{aligned}
&\frac{\partial}{\partial\theta}\left[\left(1 - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2\theta}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2\theta}\right)^2\right] \\
&= \left(\frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} - \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2}\right)\left[-\frac{2}{\theta^2} + 2\left(\frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} + \frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}\right)\frac{1}{\theta^3}\right],
\end{aligned}$$

which is negative if and only if $\theta > \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} + \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2}$ for $\theta \in [\theta_L, \theta_H]$. We are done if we show that $\theta_L > \frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} + \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2}$ for $\theta \in [\theta_L, \theta_H]$. We have $\overline{A}_2\theta_L \geq 2\hat{r}(\overline{A}_2\theta_2)$ because $\hat{r}(\overline{A}_2\theta_2) \leq 2\gamma$, and from the fact that $\hat{r}(\overline{A}_2\theta_2) > \hat{r}(\overline{A}_2\theta_1)$, we have $\overline{A}_2\theta_L > 2\hat{r}(\overline{A}_2\theta_1)$. Thus,

$$\frac{\hat{r}(\overline{A}_2\theta_2)}{\overline{A}_2} + \frac{\hat{r}(\overline{A}_2\theta_1)}{\overline{A}_2} < \frac{\theta_L}{2} + \frac{\theta_L}{2} = \theta_L,$$

which completes the proof of the claim. ■

We first show that there does not exist a separating equilibrium where only the H-type runs the project in period 1 by proving that the L-type has an incentive to mimic the H-type. Suppose that there exists such equilibrium, and let r_h be the contract the H-type offers in period 1. Note that the entrepreneur's behavior in period 2 is deterministic, in line with the findings above. If the L-type deviates by offering r_h, he/she will be treated as the H-type in period 2, unless his/her type is revealed. So the condition for the L-type not to mimic the H-type entrepreneur in period 1, i.e., deviate by offering r_h in period 1, is:

$$\begin{aligned} 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) &\geq u_1(r_h \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) \\ -\mathbf{1}_{d\in\omega}\beta \left(\frac{r_h}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_H}\right) &(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)). \end{aligned}$$

However, if cannot hold because $u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) < u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)$.

We complete the proof by showing that there does not exist a separating equilibrium where only the L-type runs the project in period 1. Suppose that there exists such equilibrium, and let r_ℓ be the contract that the L-type offers in period 1. Notice that both types have different operation histories at the beginning of period 2. Thus, by the restriction on the belief system and the following claim, $r_{i,2} = \hat{r}(\overline{A}_2\theta_i)$ for $i \in \{H, L\}$ must hold as we focus on the least-contract equilibrium. We prove the non-existence of such separating equilibrium by showing that the incentive compatibility conditions for both types in period 1 cannot hold together, i.e., at least one type of entrepreneur has the incentive to mimic the other type. If the L-type mimics the H-type, i.e., does not run the project in period 1, his/her history at the beginning of period 2 becomes the same as that of the H-type in both regimes, because this entrepreneur mimics both the operation history and the intrinsic credit history of the H-type. Thus, this entrepreneur offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2. Therefore, the incentive compatibility condition for the L-type not to mimic the H-type is:

$$u_1(r_\ell \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L). \tag{15}$$

In this equilibrium, the H-type does not run the project in period 1 and offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2. Suppose that the H-type decides to mimic the L-type to offer r_ℓ in period 1. If the default history is not observable, then the credit history of this H-type at the beginning of period 2 becomes the same as that of the L-type. Now consider that the default history is observable. Under the default history regime, the type is revealed if the credit history indicates that the entrepreneur operated the business and $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. Under the full-information regime, the type is revealed if $r_1 = r_\ell$ and $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. That is, if the H-type mimics the L-type and $d \in \omega$, then the type of the entrepreneur is revealed when $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$, thus offers $\hat{r}(\overline{A}_2\theta_H)$, according to claim 2. If $A_1 \notin \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$, then the default history for this entrepreneur coincides with that of the L-type, and both the transaction history and the operation history also coincide with that of the L-type, thus this H-type offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. The incentive compatibility condition for the H-type not to mimic the L-type is:

$$\begin{aligned} 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq{}& u_1(r_\ell \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) \\ &+ \mathbf{1}_{d\in\omega}\beta \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_\ell}{\overline{A}_1\theta_H}\right) \left(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right). \end{aligned} \tag{16}$$

We show that there cannot exist a separating equilibrium where only the L-type runs the project in period 1, by proving that both (15)and (16) cannot hold together. Suppose conversely that (15) holds along with (16) and rewrite (15) and (16) as follows, respectively:

$$\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right]\leq\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_L}\right)^2,$$

$$\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]\geq\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_H}\right)^2$$
$$+\mathbf{1}_{d\in\omega}\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right],$$

Note that

$$\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_H}\right)^2+\mathbf{1}_{d\in\omega}\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]$$
$$>\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_L}\right)^2,$$

which implies that

$$\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]>\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right].$$

However, claim 3 indicates that

$$\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right]>\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right],$$

which leads to a contradiction. ■

Proof of proposition 6. To provide robust support for the existence of this equilibrium, we assume that any deviation from the equilibrium path leads to the worst belief about the entrepreneur's type, that the entrepreneur is undoubtedly the L-type unless the type is revealed. Because the period-2 behavior of the entrepreneur is deterministic, according to claim 2 in the proof of proposition 5, it suffices to check whether each type of entrepreneur has no incentive to deviate from the equilibrium strategy in period 1. First, if the entrepreneur opts not to run the project in period 1, the operation history will reflect the inactivity. Consequently, the entrepreneur will be identified as the L-type and will end up offering $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Therefore, deviating by not running the project cannot be beneficial. Also, because any deviation in period 1 from the pooling strategy by the entrepreneur will result in being regarded as the L-type in period 1, any contract offer $r \neq \hat{r}(\overline{A}_1\theta_\sigma)$ will be accepted only if $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$. Consequently, any deviation must be offering $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1.

Let $P^i_\omega(r) \in [0,1]$ denote the probability in information regime ω that the type is perceived as the L-type in period 2 after a contract $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ is offered by i-type and is accepted in period 1. First, it is trivial that, for each $i \in \{H, L\}$, $P^i_\varnothing(r) = 0$ for all $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$, and $P^i_{\{r_1\}}(r) = 1$ for all $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ while $P^i_{\{r_1\}}(\hat{r}(\overline{A}_1\theta_\sigma)) = 0$. If $\omega = \{r_1, d\}$, then, even when $r \neq \hat{r}(\overline{A}_1\theta_\sigma)$ is observed, the default history can distinguish the type. Thus, $P^i_{\{r_1,d\}}(r) = \frac{r}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}$ for all $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ and for each $i \in \{H, L\}$. Finally, consider that $\omega = \{d\}$. If either $A_1 \geq \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}$ and $d = 0$ or $A_1 < \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}$ and $d = 1$, then the lender has a pooling belief. If $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, on the other hand, then the lender perceives the entrepreneur as the H-type if $d = 0$ and the L-type if $d = 1$. Otherwise, the lender perceives the entrepreneur as the L-type. That is, the entrepreneur who offers $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ is perceived as the L-type if either the entrepreneur is the L-type and $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{r}{\theta_L}\right)$, or the entrepreneur is the H-type and $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{r}{\theta_H}\right)$. Thus, for all $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$, $P^L_{\{d\}}(r) = \frac{r}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}$ and $P^H_{\{d\}}(r) = \frac{r}{\overline{A}_1\theta_H} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}$. Note that the H-type is treated as the H-type if $\frac{r}{\theta_H} < \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}$ and $A_1 \in \left[\frac{r}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Thus, the H-type who offers $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ faces the best belief with probability $\max\left\{0, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right\}$ and the pooling belief with probability $1 - P^H_{\{d\}}(r) - \max\left\{0, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right\}$.

We only need to show that each type does not have an incentive to deviate by offering $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1. Note that $u_1(\cdot \mid \theta_i)$ is decreasing and $P^i_\omega(\cdot)$ is weakly increasing for each $i \in \{H, L\}$. In any information regime ω, the L-type's expected utility who offers $r \in \{\hat{r}(\overline{A}_1\theta_\sigma)\} \cup [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1 is

$$u_1(r \mid \theta_L) + \beta P^L_\omega(r) u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) + \beta(1 - P^L_\omega(r)) u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)),$$

which decreases with r. Therefore, it is optimal for the L-type not to deviate from the equilibrium strategy in any information regime.

Next, we argue that the H-type would not deviate by offering $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1 when $\omega \neq \{r_1, d\}$. If either $\omega = \varnothing$ or $\omega = \{r_1\}$, then the H-type's expected utility is

$$u_1(r \mid \theta_H) + \beta P_\omega^H(r) u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) + \beta(1 - P_\omega^H(r)) u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H),$$

which decreases with r. If $\omega = \{d\}$, then the H-type's expected utility is

$$\begin{aligned}
&u_1(r \mid \theta_H) + \beta P_{\{d\}}^H(r) u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) + \beta \max\left\{0, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right\} u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \\
&+ \beta\left(1 - P_{\{d\}}^H(r) - \max\left\{0, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right\}\right) u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) \\
&= u_1(r \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) - \beta P_{\{d\}}^H(r)\left[u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right] \\
&+ \beta \max\left\{0, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right\}\left[u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)\right],
\end{aligned}$$

which decreases in r. Therefore it is optimal for the H-type to maintain the pooling strategy when $\omega \neq \{r_1, d\}$.

We complete the proof by showing that the H-type does not have an incentive to deviate from the pooling equilibrium strategy when $\omega = \{r_1, d\}$. By applying $P_{\{r_1,d\}}^H(\hat{r}(\overline{A}_1\theta_\sigma)) = \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}$, the proof is done by showing that H-type's equilibrium utility

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)),$$

is larger than the expected utility of the H-type who deviates by offering $r \in \left(\hat{r}(\overline{A}_1\theta_L), \bar{r}_L\right]$ in period 1:

$$u_1(r \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) + \beta\frac{r}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)).$$

First, because $\overline{A}_2\theta_L \geq 4\gamma$ and $\hat{r}(\cdot)$ is decreasing, we obtain

$$
\begin{aligned}
&u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) \\
=&\frac{\overline{A}_2\theta_H}{2}\left[-\frac{2\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H} + \frac{\hat{r}(\overline{A}_2\theta_H)^2}{\overline{A}_2^2\theta_H^2} + \frac{2\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H} - \frac{\hat{r}(\overline{A}_2\theta_L)^2}{\overline{A}_2^2\theta_H^2}\right] \\
=&\hat{r}(\overline{A}_2\theta_L) - \hat{r}(\overline{A}_2\theta_H) - \frac{\hat{r}(\overline{A}_2\theta_L)^2}{2\overline{A}_2\theta_H} + \frac{\hat{r}(\overline{A}_2\theta_H)^2}{2\overline{A}_2\theta_H} \leq 2\gamma.
\end{aligned}
$$

Then, $\frac{\beta}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)\right\} < \frac{2\gamma}{\overline{A}_1\theta_L} \leq \frac{1}{2}$. Furthermore,

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) - u_1(\bar{r}_{1,L} \mid \theta_H) \\
=&(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\Big(1 - \frac{(\bar{r}_{1,L} + \hat{r}(\overline{A}_1\theta_\sigma))}{2\overline{A}_1\theta_H}\Big) \\
\geq&(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\Big(1 - \frac{(\bar{r}_{1,L} + \hat{r}(\overline{A}_1\theta_L))}{2\overline{A}_1\theta_H}\Big) = (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\Big(1 - \frac{\theta_L}{2\theta_H}\Big).
\end{aligned}
$$

Finally,

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\Big(\frac{1}{\theta_L} - \frac{1}{\theta_H}\Big)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)\right\} \\
&-u_1(\bar{r}_{1,L} \mid \theta_H) - \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) - \beta\frac{\bar{r}_{1,L}}{\overline{A}_1}\Big(\frac{1}{\theta_L} - \frac{1}{\theta_H}\Big)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right\} \\
\geq&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\Big(\frac{1}{\theta_L} - \frac{1}{\theta_H}\Big)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right\} \\
&-u_1(\bar{r}_{1,L} \mid \theta_H) - \beta\frac{\bar{r}_{1,L}}{\overline{A}_1}\Big(\frac{1}{\theta_L} - \frac{1}{\theta_H}\Big)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right\} \\
\geq&(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\Big(1 - \frac{\theta_L}{2\theta_H}\Big) - \beta\frac{(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))}{\overline{A}_1}\Big(\frac{1}{\theta_L} - \frac{1}{\theta_H}\Big)\left\{u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right\} \\
>&(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\Big(1 - \frac{\theta_L}{2\theta_H} - \frac{1}{2}\Big) > 0,
\end{aligned}
$$

which completes the proof. ∎

Proof of proposition 7. Assume that $r_1 \notin \omega$ and there exists an equilibrium in which each type offers a distinct contract in period 1. Let r_h and r_ℓ be the distinct contract offers that the H-type and the L-type make in period 1, respectively.

First suppose that $r_\ell > r_h$. Specifically, we show that the L-type has an incentive to offer r_h instead of r_ℓ if $r_\ell > r_h$. The L-type in equilibrium offers r_ℓ in period 1, and, in period 2, offers $\hat{r}(\overline{A}_2\theta_\sigma)$ if the type is not revealed and offers $\hat{r}(\overline{A}_2\theta_L)$ otherwise. Additionally, the type is revealed in period 2 only if $d \in \omega$ and $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. Thus, the expected equilibrium utility for the L-type is

$$u_1(r_\ell \mid \theta_L)+\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)-\frac{\mathbf{1}_{d\in\omega}}{\overline{A}_1}\left(\frac{r_\ell}{\theta_L}-\frac{r_h}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)-u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)). \tag{17}$$

Consider that the L-type mimics the H-type in period 1, i.e., offers r_h in period 1. Then this entrepreneur defaults when $A_1 < \frac{r_h}{\theta_L}$. If $d \notin \omega$, then this entrepreneur offers a pooling contract. If $\omega = \{d\}$, on the other hand, the type is revealed in period 2 only if $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. Therefore, in period 2, the L-type who mimics the H-type in period 1 offers $\hat{r}(\overline{A}_2\theta_H)$ if $A_1 \in \left[\frac{r_h}{\theta_L}, \frac{r_\ell}{\theta_L}\right)$, offers $\hat{r}(\overline{A}_2\theta_L)$ if $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_h}{\theta_L}\right)$, and offers $\hat{r}(\overline{A}_2\theta_\sigma)$ otherwise. Thus, the expected payoff for the L-type who mimics the H-type is

$$\begin{aligned}
&u_1(r_h \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) + \frac{\mathbf{1}_{d\in\omega}}{\overline{A}_1}\left(\frac{r_\ell}{\theta_L}-\frac{r_h}{\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&- \frac{\mathbf{1}_{d\in\omega}}{\overline{A}_1}\left(\frac{r_h}{\theta_L}-\frac{r_h}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)).
\end{aligned} \tag{18}$$

We show from the following expression that the L-type has an incentive to mimic the H-type in period 1 if $r_\ell > r_h$:

$$\begin{aligned}
(18) - (17) &= u_1(r_h \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_1(r_\ell \mid \theta_L) - \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) \\
&\quad + \frac{\mathbf{1}_{d\in\omega}}{\overline{A}_1}\left(\frac{r_\ell}{\theta_L}-\frac{r_h}{\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&\quad + \frac{\mathbf{1}_{d\in\omega}}{\overline{A}_1}\left(\frac{r_\ell}{\theta_L}-\frac{r_h}{\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) \\
&\geq u_1(r_h \mid \theta_L) - u_1(r_\ell \mid \theta_L) > 0.
\end{aligned}$$

Finally, suppose that $r_\ell < r_h$. Then the H-type can increase the expected utility by offering r_ℓ in period 1 and strategically defaulting whenever $A_1 < \frac{r_h}{\theta_H}$, i.e., as if he/she offered r_h: such deviation enlarges the utility in period 1, while maintaining the utility in period 2 as in the equilibrium. ■

Proof of proposition 8. We begin by showing that $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$. First, based on the lender 1's rationality, it is not possible for $r_{L,1}$ to be less than $\hat{r}(\overline{A}_1\theta_L)$. Additionally, considering that $\hat{r}(\overline{A}_1\theta_L)$ would be accepted regardless of lender 1's belief, there is no rationale for the L-type to offer above $\hat{r}(\overline{A}_1\theta_L)$ in period 1. Thus, $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$ holds.

Next, we show that $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$, and $r_{H,1} = r_H^*$ if $d \in \omega$ and $r_{H_1} = r_H^{**}$ if $d \notin \omega$. We prove this through the L-type's incentive compatibility constraint. If the L-type mimics the H-type, i.e., deviates by offering $r_{H,1}$ in period 1, then the type will not revealed in period 2 unless $d \in \omega$ and $A_1 \in \left[\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right]$. Further, the L-type who offers $r_{H,1}$ in period 1 will offer $\hat{r}(\overline{A}_2\theta_H)$ in period 2 if the type is not revealed, and will offer $\hat{r}(\overline{A}_2\theta_L)$ in period 2 otherwise. Thus, as we define

$$F_L(r) = u_1(r \mid \theta_L)+\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)-\mathbf{1}_{d\in\omega}\beta\left(\frac{r}{\overline{A}_1\theta_L}-\frac{r}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)-u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)),$$

the L-type's incentive compatibility constraint is as follows:

$$u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq F_L(r_{H,1}).$$

It is easy to verify that F_L decreases in r. Also,

$$F_L(\hat{r}(\overline{A}_1\theta_L)) > u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),$$

which implies $F_L(\hat{r}(\overline{A}_1\theta_L)) > F_L(r_{H,1})$. Therefore, $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$. Because we restrict our attention to the least-contract equilibria, this incentive compatibility constraint for the L-type binds, given that F_L is decreasing. Thus, $r_{H,1} = r_H^*$ if $d \in \omega$ and $r_{H,1} = r_H^{**}$ if $d \notin \omega$. Thus, both $r_H^* > \hat{r}(\overline{A}_1\theta_L)$ and $r_H^{**} > \hat{r}(\overline{A}_1\theta_L)$ hold.

We finish the proof by showing $r_H^{**} > r_H^*$. According to the definition of r_H^*,

$$u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - \beta\left(\frac{r_H^*}{\overline{A}_1\theta_L}-\frac{r_H^*}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L))$$
$$= u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),$$

which implies that

$$u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) > u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L).$$

Because $u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)$ decreases in r_H^* and

$$u_1(r_H^{**} \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) = u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)$$

holds by the definition of r_H^{**}, $r_H^{**} > r_H^*$ must hold. ■

Proof of proposition 9. According to the proof of proposition 8, $(r_{H,1}, r_{L,1}) = (r_H^*, \hat{r}(\overline{A}_1\theta_L))$ holds if $\omega = \{r_1, d\}$ and $(r_{H,1}, r_{L,1}) = (r_H^{**}, \hat{r}(\overline{A}_1\theta_L))$ holds if $\omega = \{r_1\}$. To provide robust support for the existence of this equilibrium, we introduce an assumption that any deviation from the equilibrium trajectory leads to the worst belief about the entrepreneur's type, that the entrepreneur is the L-type unless the type is revealed.

Firstly, it is worth noting that the type of entrepreneur is revealed at the beginning of period 2 in the separating equilibrium. Therefore it is trivial that the H-type offers $\hat{r}(\overline{A}_2\theta_H)$, while the L-type offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2, given that we focus on the least-contract equilibria and according to claim 2 in the proof of proposition 5. It is also trivial that the L-type in such equilibrium offers $\hat{r}(\overline{A}_1\theta_L)$ in period 1. The necessary condition we need to check is that the H-type's contract offer $r_{H,1}$ in period 1 would be accepted by lender 1 who believes that such a contract is offered by the H-type entrepreneur for sure. According to the lender 1's rationality, $r_{H,1}\left(1 - \dfrac{r_{H,1}}{\overline{A}_1\theta_H}\right) \geq \gamma$ must hold, which is equivalent to $r_{H,1} \in \left[\hat{r}(\overline{A}_1\theta_H), \bar{r}_H\right]$. We already have $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$ from proposition 8, which implies $r_{H,1} > \hat{r}(\overline{A}_1\theta_H)$. Therefore, the above necessary condition eventually requires that $r_{H,1} \leq \bar{r}_H$.

To establish the conditions for the existence of a separating equilibrium without a break, it is necessary to examine whether each type of entrepreneur has an incentive to deviate from the equilibrium in period 1. A deviation by either type in period 1 would lead lender 1 to adopt the worst belief - that the entrepreneur is the L-type - unless the deviation is to mimic the other type. Consequently, if the entrepreneur deviates by offering below $\hat{r}(\overline{A}_1\theta_L)$, the contract would be rejected, precluding the running of the project in the first period.

We first focus on the H-type's incentive compatibility condition not to deviate from the equilibrium strategy. First, it is trivial that the H-type has no incentive to offer higher than $r_{H,1}$ in period 1. Now suppose that the H-type decides to deviate by offering a contract $r \in \left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$. If $\omega = \{r_1\}$, then this entrepreneur is perceived as the L-type by lender 2, thus, this entrepreneur offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. If $\omega = \{r_1, d\}$, then this entrepreneur's type is revealed if $A_1 \in \left[\frac{r}{\theta_H}, \frac{r}{\theta_L}\right)$ so that this entrepreneur offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2, and will be perceived as the L-type by the lender 2 in period 2 otherwise so that this entrepreneur offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Therefore, the H-type's expected utility who deviates by offering $r \in \left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$ in period 1 is

$$
\begin{aligned}
&= u_1(r \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) \\
&+ \mathbf{1}_{d\in\omega}\beta \cdot \left(\frac{r}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)) \equiv \hat{f}(r).
\end{aligned}
$$

Whereas, the H-type's expected equilibrium utility is

$$u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

It is trivial that $\hat{f}$ is continuous, $\hat{f}$ is convex because $\frac{\partial^2}{\partial r^2}\hat{f}(r) = \frac{\partial^2}{\partial r^2}u_1(r \mid \theta_H) = \frac{1}{\overline{A}_1\theta_H} > 0$, and

$$\hat{f}(r_{H,1}) < u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

Therefore, if $\hat{f}(\hat{r}(\overline{A}_1\theta_L)) < u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)$ holds, then $\hat{f}(r) < u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)$ is also satisfied for all $r \in \left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$. Thus, the H-type would not deviate from the equilibrium strategy whenever it is not beneficial for the H-type to mimic the L-type.

The L-type who deviates by offering a contract higher than $\hat{r}(\overline{A}_1\theta_L)$ would be unequivocally identified as the L-type in the second period unless the contract offer is $r_{H,1}$. Thus, the L-type has no incentive to deviate by offering a contract above $\hat{r}(\overline{A}_1\theta_L)$, unless he/she mimics the H-type. In conclusion, to ensure the stability of this separating equilibrium, it suffices to verify two conditions: i) each type of entrepreneur does not have an incentive to mimic the other type of entrepreneur in period 1, and ii) neither type of entrepreneur has an incentive not to run the project in period 1. From i), we need both (8) and (9) to be satisfied. Finally, the constraint for the H-type and the L-type not to break in period 1, respectively, are

$$u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)$$

and

$$u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),$$

both of which hold trivially. ■

Proof of proposition 11. First, suppose that there exists separating equilibrium, and let $r_{H,1}$ be the H-type's contract offer in period 1. Because $\overline{A}_1\theta_L < 4\gamma$ and $\overline{A}_2\theta_L < 4\gamma$, L-type earns zero payoff in this equilibrium. However, if the L-type deviates by mimicking the H-type in period 1, the type is not revealed if both types default or both types do not default, i.e., $A_1 \in \left[0, \frac{r_{H,1}}{\theta_H}\right) \cup \left[\frac{r_{H,1}}{\theta_L}, 1\right]$. That is, with probability $1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H} > 0$, the L-type who mimics the H-type in period 1 will be regarded as the H-type in period 2 by lender 2, resulting in earning a positive expected payoff by offering $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2. Therefore, separating equilibrium cannot exist.

Consider that $4\gamma > \overline{A}_1\theta_\sigma$. Then pooling equilibrium cannot exist, thus, the market collapses because separating equilibrium does not exist. Finally, $4\gamma \leq \overline{A}_1\theta_\sigma$ implies the existence of the pooling equilibrium in which both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ in period 1. ■

Proof of proposition 12. Suppose that there exists a separating equilibrium in which only one type runs the project in period 1. Then the type is revealed in period 2 regardless of the information regime. First suppose that only the L-type runs the project in period 1, and let r_ℓ be the contract that the L-type offers in period 1. In this equilibrium, the L-type offers r_ℓ in period 1 and does not run the project in period 2, while the H-type does not run the project in period 1 and offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2. The L-type's incentive compatibility constraint

$$u_1(r_\ell \mid \theta_L) \geq \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L), \tag{19}$$

and the H-type's incentive compatibility constraint

$$\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq u_1(r_\ell \mid \theta_H) + \mathbf{1}_{d\in\omega}\beta\left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_\ell}{\overline{A}_1\theta_H}\right) \cdot u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \tag{20}$$

must hold for the existence of the equilibrium. Both incentive compatibility constraints imply that

$$u_1(r_\ell \mid \theta_H) - u_1(r_\ell \mid \theta_L) \le \beta[u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)],$$

We prove that both incentive compatibility constraints cannot be held together by showing that $u_1(r_\ell \mid \theta_H) - u_1(r_\ell \mid \theta_L) > \beta[u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)]$. First, it is easy to verify that $u_i(r \mid \theta_H) - u_i(r \mid \theta_L) = \overline{A}_i\left(\frac{\theta_H}{2} - \frac{\theta_L}{2}\right) - \frac{1}{\overline{A}_i}\left(\frac{1}{2\theta_L} - \frac{1}{2\theta_H}\right) r^2$ decreases with r and increases with $\overline{A}_i$. Therefore, because $r_\ell \ge \hat{r}(\overline{A}_1\theta_L) > \hat{r}(\overline{A}_1\theta_H)$, we deduct $u_1(r_\ell \mid \theta_H) - u_1(r_\ell \mid \theta_L) > \beta\left[u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_H) - u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_L)\right] > \beta\left[u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)\right]$, a contradiction.

Now we show that there does not exist a separating equilibrium where only the H-type runs the project in period 1. In such equilibrium, if exists, the H-type offers $\hat{r}(\overline{A}_i\theta_H)$ in both periods i, while the L-type does not run a business in period 1 and cannot run a business in period 2. Thus, the L-type has an incentive to mimic the H-type. ■

Proof of proposition 13. Consider a pooling equilibrium: both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$ in period 1. There is no reason for either type to deviate by not making an offer, because such entrepreneur's operation history will indicate not having run the business in period 1, eventually resulting in the entrepreneur being regarded as the L-type for sure. Any deviation in period 1 is regarded as the L-type's behavior, thus any contract offer $r \ne \hat{r}(\overline{A}_1\theta_\sigma)$ will be accepted only if $r \in \left[\hat{r}(\overline{A}_1\theta_L), \bar{r}_{1,L}\right]$, which is larger than $\hat{r}(\overline{A}_1\theta_\sigma)$. That is, any deviation in period 1 results in a decrease in the expected utility in period 1. Thus, a deviation is not beneficial for either type if the type is not revealed after period 1. Moreover, for the L-type, the type revelation is not beneficial in period 2. Thus, there is no incentive for the L-type to deviate.

For the H-type, if $d \notin \omega$, the type cannot be revealed as the H-type. Also, if $\omega = \{d\}$, then a deviation in period 1 does not change the probability of type revelation. So we only need to check whether the H-type has an incentive to deviate when $\omega = \{r_1, d\}$. We first show that the H-type does not have such an incentive when the market does not collapse in period 2. If both types can offer the pooling contract in period 2 when the type is not disclosed, then $\overline{A}_2\theta_\sigma \ge 4\gamma$ and $r_{H,2} = \hat{r}(\overline{A}_2\theta_\sigma)$. If only the H-type makes an offer in period 2 when the type is not disclosed, then $r_{H,2} = \max\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\}$. In either case, the H-type's equilibrium utility in the pooling equilibrium is

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(r_{H,2} \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)),$$

and the expected utility of the H-type who deviates by offering $r \in \left[\hat{r}(\overline{A}_1\theta_L), \bar{r}_{1,L}\right]$ is

$$u_1(r \mid \theta_H) + \beta\frac{r}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

Because the above expression is convex in r, it suffices to check whether the H-type has the incentive to deviate to offer $\bar{r}_{1,L}$ in period 1. That is, the H-type does not have the incentive to deviate in period 1 if and only if

$$\begin{aligned}&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(r_{H,2} \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H))\\ \geq &u_1(\bar{r}_{1,L} \mid \theta_H) + \beta\frac{\bar{r}_{1,L}}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).\end{aligned}$$

We show that the above inequality always holds. We first argue that

$$\frac{\beta}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\} < \beta\left(1 - \frac{\theta_L}{2\theta_H}\right) \tag{21}$$

holds if the market does not collapse in period 2. First, if only the H-type makes an offer in period 2 when the type is not disclosed, then $u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H) = 0$ if $r_{H,2} = \hat{r}(\overline{A}_2\theta_H)$, and if $r_{H,2} = \overline{A}_2\theta_L$, then $\overline{A}_2\theta_L \geq \hat{r}(\overline{A}_2\theta_H)$, thus,

$$\begin{aligned}&u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H) = u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\overline{A}_2\theta_L \mid \theta_H)\\ =&\overline{A}_2\theta_L - \hat{r}(\overline{A}_2\theta_H) - \frac{\overline{A}_2^2\theta_L^2}{2\overline{A}_2\theta_H} + \frac{\hat{r}(\overline{A}_2\theta_H)^2}{2\overline{A}_2\theta_H} < \overline{A}_2\theta_L - \frac{\overline{A}_2^2\theta_L^2}{2\overline{A}_2\theta_H} = \overline{A}_2\theta_L - \frac{\overline{A}_2\theta_L^2}{2\theta_H}.\end{aligned}$$

As a result, $\frac{\beta}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\} < \frac{\beta}{\overline{A}_1\theta_L}\left(\overline{A}_2\theta_L - \frac{\overline{A}_2\theta_L^2}{2\theta_H}\right) < \beta\left(1 - \frac{\theta_L}{2\theta_H}\right)$, because $\overline{A}_2 < \overline{A}_1$. Next, if both types can offer the pooling contract in period 2 when the type is not disclosed, then, from the fact that $\overline{A}_2\theta_\sigma \geq 4\gamma$ and $\hat{r}(\cdot)$ is decreasing, $u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) = \hat{r}\overline{A}_2\theta_\sigma) - \hat{r}(\overline{A}_2\theta_H) - \frac{\hat{r}(\overline{A}_2\theta_\sigma)^2}{2\overline{A}_2\theta_H} + \frac{\hat{r}(\overline{A}_2\theta_H)^2}{2\overline{A}_2\theta_H} < \hat{r}\overline{A}_2\theta_\sigma) \leq 2\gamma$. Therefore $\frac{\beta}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)\ \right\} <$

$\frac{2\gamma\beta}{\overline{A}_1\theta_L} \leq \frac{\beta}{2} < \beta\left(1 - \frac{\theta_L}{2\theta_H}\right)$. We also argue that

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) - u_1(\bar{r}_{1,L} \mid \theta_H) = (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\left(1 - \frac{(\bar{r}_{1,L} + \hat{r}(\overline{A}_1\theta_\sigma))}{2\overline{A}_1\theta_H}\right)$$
$$\geq (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\left(1 - \frac{(\bar{r}_{1,L} + \hat{r}(\overline{A}_1\theta_L))}{2\overline{A}_1\theta_H}\right) = (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\left(1 - \frac{\theta_L}{2\theta_H}\right).$$

Therefore,

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(r_{H,2} \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\}$$
$$-u_1(\bar{r}_{1,L} \mid \theta_H) - \beta\frac{\bar{r}_{1,L}}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)$$
$$> u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\}$$
$$-u_1(\bar{r}_{1,L} \mid \theta_H) - \beta\frac{\bar{r}_{1,L}}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\}$$
$$\geq (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\left(1 - \frac{\theta_L}{2\theta_H}\right) - \beta\frac{(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\ u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(r_{H,2} \mid \theta_H)\ \right\}$$
$$> (\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma))\left(1 - \frac{\theta_L}{2\theta_H} - \beta + \frac{\beta\theta_L}{2\theta_H}\right) > 0.$$

Finally, suppose that the market collapses in period 2 when the type is not disclosed. The H-type's equilibrium utility in the pooling equilibrium is

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H),$$

and the expected utility of the H-type who deviates by offering $r \in \left[\hat{r}(\overline{A}_1\theta_L), \bar{r}_{1,L}\right]$ is

$$u_1(r \mid \theta_H) + \beta\frac{r}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

Because the above expression is convex in r, it suffices to check whether the H-type has the incentive to deviate to offer $\bar{r}_L$ in period 1. That is, the H-type does not have the incentive to deviate in period 1 if and only if

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) - u_1(\bar{r}_{1,L} \mid \theta_H) - \beta\frac{\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)$$
$$= \left(\bar{r}_{1,L} - \hat{r}(\overline{A}_1\theta_\sigma)\right)\left[1 - \frac{\bar{r}_{1,L} + \hat{r}(\overline{A}_1\theta_\sigma)}{2\overline{A}_1\theta_H} - \beta\frac{\overline{A}_2}{2\overline{A}_1}\left(\frac{\theta_H}{\theta_L} - 1\right)\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2\right] \geq 0.$$

Based on the fact that $\overline{A}_2 \in \left[\frac{4\gamma}{\theta_H}, \frac{4\gamma}{\theta_L}\right)$, define $f : \left[\frac{4\gamma}{\theta_H}, \frac{4\gamma}{\theta_L}\right] \to \mathbb{R}$ as

$$f(\overline{A}_2) = \left[1 - \frac{\bar{r}_{L,1} + \hat{r}(\overline{A}_1\theta_\sigma)}{2\overline{A}_1\theta_H} - \beta\frac{\overline{A}_2}{2\overline{A}_1}\left(\frac{\theta_H}{\theta_L} - 1\right)\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2\right].$$

It is trivial that f is decreasing. Because $\frac{\bar{r}_{L,1}+\hat{r}(\overline{A}_1\theta_\sigma)}{2\overline{A}_1\theta_H} < \frac{\bar{r}_{L,1}+\hat{r}(\overline{A}_1\theta_L)}{2\overline{A}_1\theta_H} = \frac{\theta_L}{2\theta_H} < \frac{1}{2}$, $\overline{A}_1\theta_L \geq 4\gamma$, and $\hat{r}(4\gamma) = \frac{1}{2}$,

$$f\left(\frac{4\gamma}{\theta_H}\right) = 1 - \frac{\bar{r}_{L,1} + \hat{r}(\overline{A}_1\theta_\sigma)}{2\overline{A}_1\theta_H} - \beta\frac{4\gamma}{2\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left(1 - \frac{\hat{r}(4\gamma)}{4\gamma}\right)^2 > \frac{1}{2} - \frac{\beta}{4}\frac{4\gamma}{2\overline{A}_1\theta_L} > \frac{1}{2} - \frac{\beta}{8} > 0.$$

Moreover,

$$f\left(\frac{4\gamma}{\theta_L}\right) = 1 - \frac{\bar{r}_{L,1} + \hat{r}(\overline{A}_1\theta_\sigma)}{2\overline{A}_1\theta_H} - \beta\frac{4\gamma}{2\overline{A}_1\theta_L}\left(\frac{\theta_H}{\theta_L} - 1\right)\left(1 - \frac{\hat{r}\cdot\left(4\gamma\frac{\theta_H}{\theta_L}\right)}{4\gamma\cdot\frac{\theta_H}{\theta_L}}\right)^2$$

Let $\hat{A}_2$ be such that $f(\hat{A}_2) = 0$,[26] and define $\overline{A}_2^* \equiv \min\left\{\frac{4\gamma}{\theta_L}, \hat{A}_2\right\}$. $\hat{A}_2 > \frac{4\gamma}{\theta_H}$ is satisfied because $f\left(\frac{4\gamma}{\theta_H}\right) > 0$, thus, $\overline{A}_2^* \in \left(\frac{4\gamma}{\theta_H}, \frac{4\gamma}{\theta_L}\right]$. Note that $f(\overline{A}_2) \geq 0$ if and only if $\overline{A}_2 \leq \overline{A}_2$. Therefore, pooling equilibrium without a break exists if and only if $\overline{A}_2 \leq \overline{A}_2^*$. ■

Proof of proposition 15. Consider an equilibrium in which each type offers a distinct contract in period 1. Then, only the H-type offers, $\hat{r}(\overline{A}_2\theta_H)$, in period 2. Also, either $\omega = \{r_1\}$ or $\omega = \{r_1, d\}$ is required for the existence of such equilibrium.

[26] $f\left(\frac{4\gamma}{\theta_L}\right)$ can be either positive or negative because $\lim_{\theta_H\to\theta_L} f\left(\frac{4\gamma}{\theta_L}\right) > 0$ and $\lim_{\theta_H\to\infty} f\left(\frac{4\gamma}{\theta_L}\right) < 0$. Therefore $\hat{A}_2 > \frac{4\gamma}{\theta_L}$ if $f\left(\frac{4\gamma}{\theta_L}\right) > 0$ and vice versa.

The incentive compatibility constraint for both types, respectively, are

$$u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_H), \tag{22}$$

$$u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) \geq u_1(r_{H,1} \mid \theta_L) + \beta \left\{1 - \mathbf{1}_{\{\omega=\{r_1,d\}\}} \left(\frac{r_{H,1}}{\overline{A}_1\theta_L} - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\right\} u_2\left(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L\right). \tag{23}$$

According to proposition 14, (23) binds. Thus, as we re-write (22) and (23), we obtain

$$\left(1 - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)^2 - \left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)^2 \leq \beta\frac{\overline{A}_2}{\overline{A}_1}\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2, \tag{24}$$

$$\left(1 - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}\right)^2 - \left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L}\right)^2 = \beta\left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\frac{\overline{A}_2}{\overline{A}_1}\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2. \tag{25}$$

By claim 3 in the proof of proposition 5, we have

$$\left(1 - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}\right)^2 - \left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L}\right)^2 > \left(1 - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)^2 - \left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)^2.$$

Also, it is trivial that

$$\frac{\overline{A}_2}{\overline{A}_1}\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2 > \left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\frac{\overline{A}_2}{\overline{A}_1}\left(1 - \frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2.$$

Therefore, if (25) binds then (24) also holds. Finally, the lender's incentive compatibility constraint requires that $r_{H,1} \in [\hat{r}(\overline{A}_1\theta_H), \bar{r}_{1,H}]$. (25) implies $r_{H,1} \geq \hat{r}(\overline{A}_1\theta_L)$, thus, the lender's incentive compatibility condition requires $r_{H,1} \leq \bar{r}_{1,H}$, which completes the proof. ■

Proof of lemma 1. We need to show that

$$\begin{aligned}
&\sigma\left[u_i(\hat{r}(\overline{A}_t\theta_\sigma)\mid\theta_H)+\hat{r}(\overline{A}_t\theta_\sigma)\Big(1-\frac{\hat{r}(\overline{A}_t\theta_\sigma)}{\overline{A}_t\theta_H}\Big)\right]+(1-\sigma)\left[u_i(\hat{r}(\overline{A}_t\theta_\sigma)\mid\theta_L)+\hat{r}(\overline{A}_t\theta_\sigma)\Big(1-\frac{\hat{r}(\overline{A}_t\theta_\sigma)}{\overline{A}_t\theta_L}\Big)\right]\\
&-\sigma\left[u_i(\hat{r}(\overline{A}_t\theta_H)\mid\theta_H)+\hat{r}(\overline{A}_t\theta_H)\Big(1-\frac{\hat{r}(\overline{A}_i\theta_H)}{\overline{A}_t\theta_H}\Big)\right]-(1-\sigma)\left[u_i(\hat{r}(\overline{A}_t\theta_L)\mid\theta_L)+\hat{r}(\overline{A}_t\theta_L)\Big(1-\frac{\hat{r}(\overline{A}_t\theta_L)}{\overline{A}_t\theta_H}\Big)\right]\\
=&\sigma\frac{\overline{A}_t\theta_H}{2}\left(1-\left(\frac{\hat{r}(\overline{A}_t\theta_\sigma)}{\overline{A}_t\theta_H}\right)^2\right)+(1-\sigma)\frac{\overline{A}_t\theta_L}{2}\left(1-\left(\frac{\hat{r}(\overline{A}_t\theta_\sigma)}{\overline{A}_t\theta_L}\right)^2\right)\\
&-\sigma\frac{\overline{A}_t\theta_H}{2}\left(1-\left(\frac{\hat{r}(\overline{A}_t\theta_H)}{\overline{A}_t\theta_H}\right)^2\right)-(1-\sigma)\frac{\overline{A}_t\theta_L}{2}\left(1-\left(\frac{\hat{r}(\overline{A}_t\theta_L)}{\overline{A}_t\theta_L}\right)^2\right).
\end{aligned}$$

is positive. As we let $p\equiv\frac{\sigma}{\overline{A}_t\theta_H}$ and $q\equiv\frac{1-\sigma}{\overline{A}_t\theta_L}$, and noticing that $p+q=\frac{1}{\overline{A}_t\theta_\sigma}$, it is equivalent to show that

$$p\left(\hat{r}\left(\frac{\sigma}{p}\right)\right)^2+q\left(\hat{r}\left(\frac{1-\sigma}{q}\right)\right)^2>p\left(\hat{r}\left(\frac{1}{p+q}\right)\right)^2+q\left(\hat{r}\left(\frac{1}{p+q}\right)\right)^2.$$

It holds if and only if $(\hat{r}(\cdot))^2$ is convex, because $p\cdot\frac{\sigma}{p}+q\cdot\frac{1-\sigma}{q}=p\cdot\frac{1}{p+q}+q\cdot\frac{1}{p+q}=1$. We first show that $\hat{r}(\cdot)$ is convex. Because $\frac{\partial\hat{r}(x)}{\partial x}=\frac{1}{2}\left[1-(x-2r)(x^2-4xr)^{-\frac{1}{2}}\right]\leq 0$, we have

$$\frac{\partial^2\hat{r}(x)}{\partial x^2}=\frac{1}{4}(x-2r)(x^2-4xr)^{-\frac{3}{2}}(2x-4r)-\frac{1}{2}(x^2-4xr)^{-\frac{1}{2}}=(x^2-4xr)^{-\frac{3}{2}}2r^2\geq 0.$$

Finally, by taking the second derivative, we obtain:

$$\frac{\partial^2\left(\hat{r}(x)\right)^2}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\left(\hat{r}(x)\right)^2}{\partial x}\right)=\frac{\partial}{\partial x}\left(2\hat{r}(x)\frac{\partial\hat{r}(x)}{\partial x}\right)=2\left(\frac{\partial\hat{r}(x)}{\partial x}\right)^2+2\hat{r}(x)\cdot\frac{\partial^2\hat{r}(x)}{\partial x^2}>0.$$

Thus, $(\hat{r}(\cdot))^2$ is also convex, which completes the proof. ■

Proof of proposition 17. Define

$$w(\theta_L,\sigma)\equiv\frac{W_{\{d\}}-W_\varnothing}{\beta}=\sigma u_2(\hat{r}(\overline{A}_2\theta_H)\mid\theta_H)-\sigma u_2(\hat{r}(\overline{A}_2\theta_\sigma)\mid\theta_H)-(1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_\sigma)\mid\theta_L),$$

which is continuous in both θ_L and σ. Also, assume that an equilibrium exists where both types offer the pooling contract in both periods when the entrepreneur's type is not disclosed, that is, $\sigma>\sigma^*$.

First, assume $\overline{A}_2\theta_L \leq 2\gamma$. Then $\sigma^* = \sigma_2$. By the definition of σ_2, we have $u_2(\hat{r}(\overline{A}_2\theta_{\sigma_2}) \mid \theta_L) = 0$, thus, $w(\theta_L, \sigma_2) > 0$. Finally, due to the continuity of w with respect to σ, either $w(\theta_L, \sigma) > 0$ for all $\sigma > \sigma_2$ or there exists $\sigma^{**} \in (\sigma_2, 1)$ such that $w(\theta_L, \sigma) > 0$ for all $\sigma \in (\sigma_2, \sigma^{**})$. As we define $\sigma^{**} = 1$ if $w(\theta_L, \sigma) > 0$ holds for all $\sigma > \sigma_2$, $W_{\{d\}} > W_\varnothing$ is satisfied whenever $\sigma \in (\sigma^*, \sigma^{**})$ for some $\sigma^{**} \in (\sigma^*, 1]$, because $\sigma_2 = \sigma^*$.

Next, suppose that $\overline{A}_2\theta_L \in (2\gamma, 4\gamma)$. Then $\sigma^* = \sigma_1$. Suppose that $w(\theta_L, \sigma_1) > 0$ for a while. Then, given the continuity of w with respect to σ, and because $\sigma_1 = \sigma^*$, it can be inferred that $W_{\{d\}} > W_\varnothing$ holds whenever $\sigma \in (\sigma^*, \sigma^{**})$ for some $\sigma^{**} \in (\sigma^*, 1]$. To wrap up the proof, we need to demonstrate the existence of a $\theta_L^* \in \left(\frac{2\gamma}{\overline{A}_2}, \frac{4\gamma}{\overline{A}_2}\right)$ such that for all $\theta_L < \theta_L^*$, $w(\theta_L, \sigma_1) > 0$ holds. If $\theta_L = \frac{2\gamma}{\overline{A}_2}$, then $\hat{r}(\overline{A}_2\theta_{\sigma_2}) = 2\gamma$, which implies $\overline{A}_2\theta_{\sigma_2} = 4\gamma$, thus, $\sigma_1 = \sigma_2$. Additionally, given that $w(\theta_L, \sigma_2) > 0$, we also have $w(\theta_L, \sigma_1) > 0$ when $\theta_L = 2\gamma$. On the other hand, if $\theta_L = 4\gamma\overline{A}_2$, then $\hat{r}(\overline{A}_2\theta_L)$ is well-defined, and we have $u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) = 0$. Thus, we have

$$\begin{aligned} w(\theta_L, \sigma_1) &= \sigma_1 u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1 - \sigma_1) u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \\ &\quad - \sigma_1 u_2(\hat{r}(\overline{A}_2\theta_{\sigma_1}) \mid \theta_H) - (1 - \sigma_1) u_2(\hat{r}(\overline{A}_2\theta_{\sigma_1}) \mid \theta_L). \end{aligned}$$

By lemma 1, we have $w(\theta_L, \sigma_1) < 0$ when $\theta_L = 4\gamma$. Finally, because $w(\cdot, \cdot)$ is continuous in θ_L, there exists $\theta_L^* \in (\frac{2\gamma}{\overline{A}_2}, \frac{4\gamma}{\overline{A}_2})$ such that $w(\theta_L, \sigma_1) > 0$ whenever $\theta_L < \theta_L^*$. ■